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STOCHASTIC AVAILABILITY OF A REPAIRABLE SYSTEM WITH AN AGE - AN--ETC(U)
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Polytechnic Institute of New York

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June 1982

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by

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Prepared for
Office of Naval Research
Contract N00014-75-C-0858

Report No. Poly EE/CS 82-004

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C O N T E N T S

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ABSTRACT

A general repairable system model is proposed with a maintenance schedule such that the failure rate is reduced after each preventive maintenance. The model introduces naturally the concept of system availabilities as random variables. Probability distributions of the stochastic availabilites are derived. The stochastic availability is optimized with respect to the duration of the operating interval between maintenance operations, and with respect to an age replacement policy. Analysis and optimization are achieved by a PASCAL computer program which is developed for computations and illustrations of various failure rate characteristics and parameter variations. The concept is further extended to stochastic cost function or generalized stochastic availability.

ILLUSTRATIONS

FIGURE

FIGURE

TABLE

FORMULAS

PROGRAM

CHAPTER 1

INTRODUCTION

§ 1.1 CONCEPTS OF SYSTEM AVAILABILITY

Reliability is a relatively new field of engineering (Shooman (1968); Dhillon and Singh (1981)) and applied probability (Barlow and Proschan (1965)). It has found its importance in the planning, design, and operation of systems in recent years. Formally, system reliability is defined as the probability that a system operates without failure during the time interval $[0,t]$, that is,

$$P\{ T > t \} = 1 - F(t) \tag{1.1.1}$$

where T is the (random) lifetime of failure-free operation of an initially good system until failure, $F(.)$ is the distribution function (d.f.) of T and $P\{.\}$ denotes the probability measure. The reliability is mainly defined for unrepairable systems. Other definitions can be found in various references (Barlow and Proschan (1965); Kozlov and Ushakov (1970)). Note that the field of reliability engineering concerns the probability of non-negative random variables only.

For repairable and maintained systems, a measure of system performance and a main design crierion is the concept of system availability. Various definitions of availability have been defined in the past three decades (Osaki and Nakagawa (1976); Lie, Hwang and Tillman (1977)). We shall review some of them.

Hasford (1960), Barlow and Hunter (1960), and also Barlow and Proschan (1965) defined the following three types of availability :

(1) Instantaneous Availability / Pointwise Availability

The instantaneous (or pointwise) availability is the probability that the system is operative at any time t. It gives a good mesure for systems functioning at any random time.

(2) Average Uptime Availability / Interval Availability

The average uptime (or interval) availability is the expected ratio of uptime in a given time interval. It is a satisfactory measure for system working over a duty cycle.

(3) Steady-State Availability / Limiting-Interval Availability

The steady-state (or limiting-interval) availability is the average uptime availability when the time interval is very large. It is relatively simple to calculate and it is a quite satisfactory measure for continously operating systems. (See also Eq.(1.1.3)).

Kabak (1969) proposed the following two versions of availability for systems with up and down cycles for an exponential failure time and constant repair time :

(a) Availability for Multipe Cycles

The availability of n cycles ($n = 1,2,3,\ldots$) is the expected value of the proportion of total uptime in the n cycles to the total elapsed time in the n cycles.

(b) Finite Time Availability

The finite time availability $A(T)$ for a time interval $[0,T]$ is determined by combining the probability of n failures in a given time interval with the proportion of available time for the interval. (See Kabak (1969) for detail).

It has been shown by Kabak (1969) that for exponential failure time and constant repair time,

(a) as the number of cycles increases, the limit of the availability for multiple cycles is the steady-state availability, and

(b) as the time interval increases the finite time availability also approaches the steady-state availability.

Consider a repairable system with up and down cycles all of which are independent. Let

$T_{up} \equiv$ uptime random variable in an up-cycle

$T_{down} \equiv$ downtime random variable in a down-cycle

Note that all the T_{up}'s are independent and identically distributed (i.i.d.) random variables and all the T_{down}'s are also i.i.d. It can be shown by renewal theory for an alternating renewal process (Parzen (1962)) that

$$\lim_{t\to\infty} P\{\text{System is up at } t\} = \frac{E[T_{up}]}{E[T_{up}] + E[T_{down}]} \tag{1.1.2}$$

$$= \frac{MTBF}{MTBF + MTTR} \tag{1.1.2'}$$

where $E[\cdot]$ = expectation

MTBF = mean time between failure

MTTR = mean time to repair.

The fraction in Eq.(1.1.2) is simpy the <u>steady-state availability</u> which will be denoted by

$$A^{\infty}$$

Thus,

$$A^{\infty} \equiv \frac{E[\text{ uptime }]}{E[\text{ uptime }] + E[\text{ downtime }]} \tag{1.1.3}$$

In particular, for exponentially distributed uptime and downtime, namely, with probability density functions (p.d.f.)

$$f_{T_{up}}(t) = \lambda \exp(-\lambda t) , \qquad t \geq 0 \tag{1.1.4}$$

and

$$f_{T_{down}}(t) = \mu \exp(-\mu t) , \qquad t \geq 0 \tag{1.1.5}$$

then

$$A^{\infty} = \frac{1/\lambda}{1/\lambda + 1/\mu} = \frac{\mu}{\mu + \lambda} \tag{1.1.6}$$

Eq.(1.1.3) is widely accepted as the (simplest and non-random) definition of system availability. However Eq.(1.1.3) gives only the average value and there is no probabilistic guarantee that A^{∞} will ever be achieved. A partial remedy had been proposed by Martz (1971) who defined the following random variable :

$$A \equiv \frac{\text{uptime}}{\text{uptime + downtime}} \tag{1.1.7}$$

$$= \frac{T_{up}}{T_{up} + T_{down}} \tag{1.1.7'}$$

Since T_{up} and T_{down} are random variables, A is indeed a random variable. Martz studied the following definition of availability :

Single Cycle Availability

The single cycle availability A_r is the value such that

$$P\{A \geq A_r\} = r \tag{1.1.8}$$

for all $r \in [0,1]$, or

$$\int_{A_r} f_A(a)\, da = r \tag{1.1.8'}$$

where $f_A(.)$ is the p.d.f. of A .

By specifying r , Martz's definition gives a probabilistic guarantee on the frequency of occurrence of the availability value A_r . Martz also derived some expressions of A_r by evaluating $f_A(.)$ for independent uptime and downtime. In particular, if T_{up} and T_{down} are exponential as given in Eq.(1.1.4) and (1.1.5), then

$$A_r = \frac{(1-r)\mu}{(1-r)\mu + r\lambda} \tag{1.1.9}$$

When $r=\frac{1}{2}$,

$$A_{\frac{1}{2}} = A^{\infty} \tag{1.1.10}$$

Nakagawa and Goel (1973) extended Martz's definition to a finite time interval :

Availability for a Finite Interval

Let

$$A(t) \equiv \frac{T_{up}}{T_{up} + T_{down}} \equiv \frac{T_{up}}{t} \tag{1.1.11}$$

The availability for a finite interval $A_r(t)$ is such that for $t \geq 0$ and $r \in [0,1]$,

$$P\left\{A(t) \geq A_r(t)\right\} = r \tag{1.1.12}$$

Some complicated expressions for $A_r(t)$ had been derived by them for independent uptime and downtime.

Marshall and Goldstein (1980) studied a repairable system and introduced the concept of cycle availability as a random variable :

Cycle Availability

The cycle availability A_c is a random variable defined as

$$A_c \equiv \frac{\text{Total uptime for a complete cycle}}{\begin{pmatrix}\text{Total uptime for}\\ \text{a complete cycle}\end{pmatrix} + \begin{pmatrix}\text{Total downtime for}\\ \text{a complete cycle}\end{pmatrix}} \tag{1.1.13}$$

where a complete cycle of the system begins with a new operative system and ends until failure, followed by a renewal or complete repair.

Goldstein considered system designs and studied the following probabilistic inequality

$$P\left\{A_c \geqslant 1-\epsilon\right\} \geqslant 1-\delta \tag{1.1.14}$$

where $0<\epsilon<1$ and $0<\delta<1$. He derived some expressions for repairable systems with independent exponential lifetime and exponential repair time, so that the technique of Laplace transform was applicable to give an Erlang distribution (Parzen (1962)). However no numerical illustrations were given. The generalization of independent exponential distribution to more general distributions, such as Weibull distribution (see Appendix 1), is not straight forward, because the Laplace transform technique does not give a closed form solution.

In this research, various concepts of system availability are studied as random variables, and they will be called stochastic availabilities, in order to emphasize their random nature. In particular, Goldstein's model and approach are extended to a general repairable system with maintenance schedule such that the failure rate is both age and maintenance dependent.

§ 1.2 MAINTENANCE AND FAILURE RATE

As we have seen so far, the various definitions and expressions of system availability have no explicit dependence on the maintenance or repairs. Most authors have treated failure and repair as independent random variables, neither of which depends on the maintenance. In other words, the system failure rate is not affected by any preventive maintenance. The use of failure rate seems to be a natural way of system failure analysis because a complete knowledge is obtained once we know or specify the failure rate (see Theorem 2.2.1).

The purpose of preventive maintenance for repairable system is most likely to improve the system if possible. However the failure rate may be disturbed by the number of maintenance operations. Various replacement and maintenance policies had been proposed (Barlow and Proschan (1965); Jorgenson, McCall and Radner (1967); and Pierskalla and Voelker (1976)). Basically, an optimum replacement policy is to choose a set of time intervals between two maintenance periods at which replacements are to take place such as to minimize an expected cost during a given finite or infinite time. Note that the cost is a deterministic quantity. Recently, Nguyen and Murthy (1981) have extended Barlow and Hunter's (1960) two replacement policies to a case where the failure rate is an increasing function with the number of repairs. A further generalization of the optimization problem to failure rate dependence on the number of previous repairs and on the times when they took place was proposed by Shaw, Ebrahimian and Chan (1981).

As with availability, the cost to be minimized can be generalized to a random variable approach by modifying the definitions of stochastic availability to be associated with costs, so that minimizing the probability of the so called stochastic cost is equivalent to maximizing that of the stochastic availability (see Chapter 5).

§ 1.3 SUMMARY

In this research various concepts of stochastic availability for a general repairable system model with a maintenance schedule and an age and maintenance dependent failure rate are studied and are applied to system design. In Chapter 2, the general system model is proposed. The system failure rate is reduced and altered after each preventive maintenance. Two types of failure reduction are considered. The lifetime distributions for both failure reduction types are derived. In Chapter 3, various stochastic availabilities are defined. The probability distributions of the stochastic availabilities are derived. In Chapter 4, optimum system designs are considered using the concept of maximizing the probability of stochastic availability and probabilistic inequality. In particular, optimum operative-cycle time and age replacement time are determined for a periodic operative-maintenance policy. Numerical examples illustrate the failure rate characteristics and variations of system parameters and age replacement time. A PASCAL computer program is developed for the computation of the probability distributions of stochastic availabilities. In Chapter 5, the concept is further extended to the so called generalized stochastic availability and the equivalent stochastic cost function in probabilistic optimization. In Chapter 6, the work is concluded with a discussion on possible extensions. Appendix 1 gives a summary of Weibull distribution. Appendix 2 is a listing of the PASCAL computer program.

CHAPTER 2

SYSTEM MODEL

§ 2.1 A REPAIRABLE SYSTEM WITH MAINTENANCE SCHEDULE

We shall study a repairable syttem with a maintenance schedule as shown in Fig. 2.1.1.

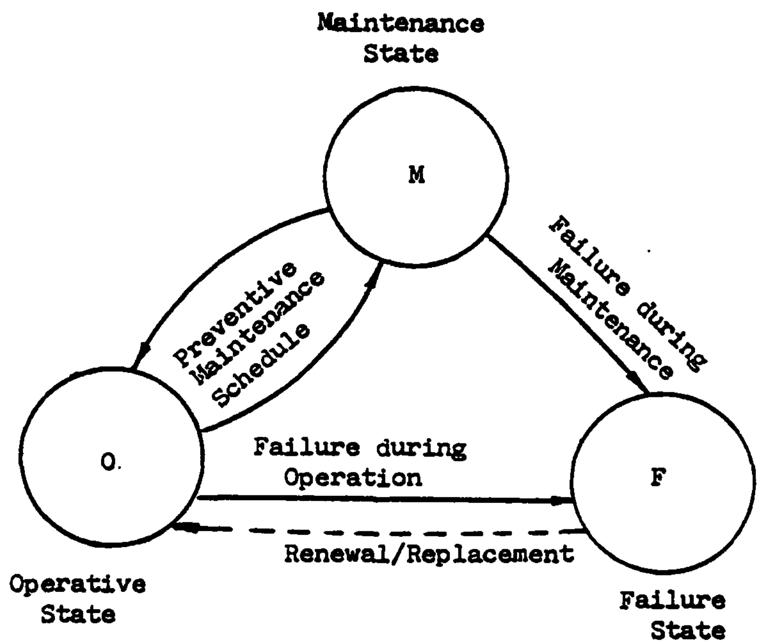


Fig. 2.1.1 State transition diagram of a repairable system with a maintenance schedule.

The system starts its mission when it is new and it follows an operative and maintenance schedule before it fails. Thus the system is represented by an UP operative state (O) of normal operation, a DOWN maintenance state (M) of preventive maintenance. By preventive maintenance we mean that the system is not replaced by a new one completely, but simply a replacement of parts, minors repairs or even just inspections. A DOWN failure state (F) accounts for the system failure or breakdown. A new system is set to operate for a time period called the operative-cycle time (u) and the system is said to be in the operative-cycle (or simply, the O-cycle). Then it is brought to preventive maintenance for a duration called the maintenance-cycle time (d) and the system is in the maintenance-cycle (the M-cycle). After the maintenance operation the system is back to normal operation. The transition

$$\text{(O)} \rightleftharpoons \text{(M)}$$

is called an operative-maintenance cycle (an O-M cycle). The system may fail during the O-cycle or during the M-cycle. When the system breaks down, it is in the failure state (F) , and it will only resume its normal operation as a new system after a complete repair, renewal or replacement of the entire system. The time spent for such a renewal is assumed to be a random variable called the repair time T_F . If the transition

$$\text{(F)} \longrightarrow \text{(O)}$$

is completed, the system is said to have gone through a complete cycle and the entire system is considered to be a new system again.

A schedule of the O-M cycles, operative-cycle time and Maintenance-cycle time is shown in Fig. 2.1.2.

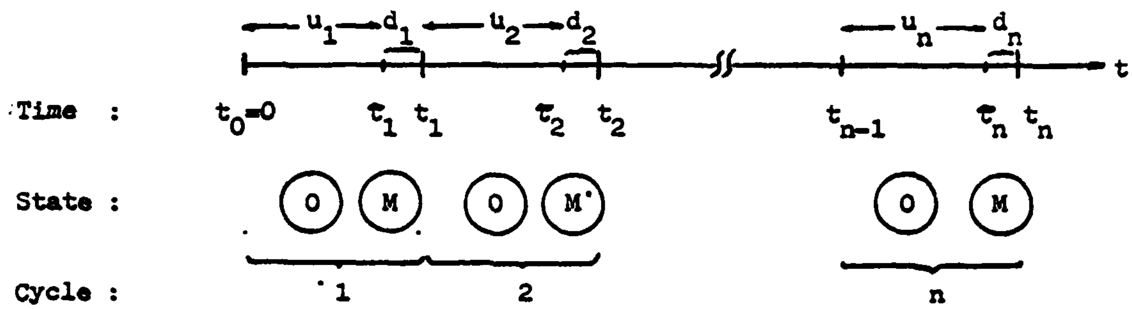


Fig. 2.1.2 System operative-maintenance schedule before failure.

We have used the following notations in Fig. 2.1.2 :

$t_0 \equiv 0$

O-Cycles : $[t_0, \tau_1)$, $[t_1, \tau_2)$, ... , $[t_{n-1}, \tau_n)$, ...

M-Cycles : $[\tau_1, t_1)$, $[\tau_2, t_2)$, ... , $[\tau_n , t_n)$, ...

O-M Cycles : $[t_0, t_1)$, $[t_1, t_2)$, ... , $[t_{n-1}, t_n)$, ...

Operative-cycle time : $u_1 = \tau_1 - t_0$, ... , $u_n = \tau_n - t_{n-1}$, ...

Maintenance-cycle time : $d_1 = t_1 - \tau_1$, ... , $d_n = t_n - \tau_n$, ...

In particular, we shall mainly interest in the periodic operative-maintenance schedule (see also § 4.1), namely,

$$\left.\begin{aligned} u_1 = u_2 = \ldots = u_n = \ldots &\equiv u \\ d_1 = d_2 = \ldots = d_n = \ldots &\equiv d \\ t_n &= n.t_1 \\ t_1 &= u + d \end{aligned}\right\} \quad (2.1.1)$$

NOTE : All time variables (deterministic or random) are assumed non-negative. Therefore, all probability density functions and failure rate functions are defined to be zero for any negative time. We shall not specify this assumption explicitly in all expressions.

§ 2.2 FAILURE RATE WITH AGE AND MAINTENANCE DEPENDENCE

The lifetime of the repairable system before failure is a random variable T with a distribution function (d.f.) $F(.)$ known as the failure distribution function (Barlow and Proschan (1965)). Usually, $F(.)$ is assumed to be absolutely continuous so that there exists a probability density function (p.d.f.) given by

$$f(t) = \frac{dF(t)}{dt} \tag{2.2.1}$$

or,

$$F(t) = \int_0^t f(x)\,dx \tag{2.2.2}$$

Note that

$$f(t) = 0 \quad \text{for} \quad t < 0$$

and

$$F(t) = 0 \quad \text{for} \quad t < 0$$

The system failure rate function (f.r.) or the system hazard rate function $r(.)$ is defined by

$$r(t) \equiv \frac{f(t)}{1 - F(t)} \tag{2.2.3}$$

$$= \frac{-\frac{dR(t)}{dt}}{R(t)} \tag{2.2.4}$$

where

$$R(t) \equiv 1 - F(t) \equiv \overline{F(t)} \tag{2.2.5}$$

is called the system reliability function (r.f.).

From Eq.(2.2.4), we have

$$\frac{dR(t)}{dt} + r(t)R(t) = 0$$

Solving,

$$\left.\begin{aligned} R(t) &= 1 - F(t) = \exp\left\{-\int_0^t r(x)\,dx\right\} \\ R(0) &= 1 \\ F(0) &= 0 \end{aligned}\right\} \quad (2.2.6)$$

Eq.(2.2.6) and (2.2.3) give

$$f(t) = r(t)R(t) \quad (2.2.7)$$

or

$$f(t) = r(t).\exp\left\{-\int_0^t r(x)\,dx\right\} \quad (2.2.7')$$

Eq.(2.2.7') is of fundamental importance.

In order to be a valid failure rate function, $r(t)$ must satisfy the following conditions :

$$\left.\begin{aligned} r(t) &= 0 \quad , \quad t < 0 \\ r(t) &\geq 0 \quad , \quad t \geq 0 \\ \int_0^{\infty} r(x)dx &= \infty \end{aligned}\right\} \quad (2.2.8)$$

From Eq.(2.2.2), (2.2.6) and (2.2.7'), we see that $F(.)$, $f(.)$, and $r(.)$ are equally suitable for describing the failure distribution. In particular, if we specify the failure rate, then the following theorem is obvious :

<u>THEOREM 2.2.1</u>

A failure rate function $r(.)$ satisfying (2.2.8) uniquely determines the failure distribution of the system.

Since there are infinitely many failure laws, we shall choose a widely useful one, namely, the Weibull type failure rate (Appendix 1), that is, the system lifetime T has a Weibull distribution :

$$\text{P.d.f.} \qquad \overset{o}{f}(t) = \lambda^{\alpha}\alpha t^{\alpha-1}\exp(-\lambda^{\alpha}t^{\alpha}) \tag{2.2.9}$$

$$\text{D.f.} \qquad \overset{o}{F}(t) = 1 - \exp(-\lambda^{\alpha}t^{\alpha}) \tag{2.2.10}$$

$$\text{F.r.} \qquad \overset{o}{r}(t) = \lambda^{\alpha}\alpha t^{\alpha-1} \tag{2.2.11}$$

$$\text{R.f.} \qquad \overset{o}{R}(t) = \exp(-\lambda^{\alpha}t^{\alpha}) \tag{2.2.12}$$

In particular,

(1) $\alpha = 1$ (exponential case), constant failure rate;

(2) $\alpha = 2$ (Rayleigh case), linear failure rate.

Therefore, the failure rate is <u>age dependent</u> whenever $\alpha \neq 1$.
We shall study the Rayleigh case ($\alpha = 2$), or the linear failure rate, because all other cases can be analyzed in the same way.

In order to allow the failure rate $r(t)$ to be dependent on the maintenance operations, the failure rate is assumed to change after each maintenance-cycle. Thus the failure rate is piecewisely defined on each O-M cycle in the following manner :

$$r(t) \equiv \begin{cases} r_1(t) , & t \in [t_0, t_1) \\ r_2(t) , & t \in [t_1, t_2) \\ \vdots & \\ r_n(t) , & t \in [t_{n-1}, t_n) \\ \vdots & \end{cases} \tag{2.2.13}$$

where

$$r_1(t),\ r_2(t),\ \dots\ ,\ r_n(t),\ \dots$$

is a sequence of Weibull type failure rates subjected to (2.2.8) , namely,

$$r_1(t) \geq 0 \quad , \quad r_2(t) \geq 0 \quad , \quad \ldots \quad , \quad r_n(t) \geq 0 \quad , \quad \ldots$$

$$\int_{t_0}^{t_1} r_1(x)dx + \int_{t_1}^{t_2} r_2(x)dx + \ldots + \int_{t_{n-1}}^{t_n} r_n(x)dx + \ldots = \infty \qquad (2.2.14)$$

The following form of the failure rate is proposed :

For $n = 1,2,3, \ldots$

$$r_n(t) = \overset{o}{r}(t;\lambda_n,\alpha) - \Delta(t) \quad , \quad t \in [t_{n-1},t_n) \qquad (2.2.15)$$

where

$$\overset{o}{r}(t;\lambda_n,\alpha) = \lambda_n^{\alpha} \alpha t^{\alpha-1} \qquad (2.2.16)$$

$$0 < \lambda = \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \ \ldots \leq \lambda_n \leq \ldots < \infty \qquad (2.2.17)$$

$$\Delta(t) = \Delta_n \quad , \quad t \in [t_{n-1},t_n) \qquad (2.2.18)$$

$$0 = \Delta_1 \leq \Delta_2 \leq \ldots \leq \Delta_n \leq \ldots . \qquad (2.2.19)$$

$$\Delta_n = \Delta_{n-1} + g_n \cdot r_{n-1}(t_{n-1}) \qquad (2.2.20)$$

$$g_1 = 0 \ , \quad g_n \in [0,1] \quad , \ n = 2,3,\ldots \qquad (2.2.21)$$

The failure rate $r(t)$ so defined has discontinuities of "jump" type at a countable set of points

$$t_0 , t_1 , \ldots , t_n , \ldots$$

and $\Delta(t)$ also has discontinuities at the same points representing a reduction failure rate. Obviously, if there is no failure reduction, then

$$\Delta(t) = 0 \quad \Longrightarrow \quad r(t) = r(t;\lambda,\alpha)$$

Thus the benefit of the maintenance schedule improves an aging system in the sense that the system failure rate is reduced after each O-M cycle with a tradeoff of shortening the mean lifetime because the λ's which is a measure of mean lifetime (Appendix 1, Eq.(A1.4)), increase with the number of O-M cycles. For convenience, we define the following :

$\overset{\circ}{r}(t;\lambda,\alpha)$ = regular failure rate ($\equiv \overset{\circ}{r}(t)$ if λ and α are constants)

$r(t)$ = age and maintenance dependent failure rate

$\Delta(t)$ = failure reduction function

g_n = failure reduction factor

Δ_n = reduction jump-down

As an illustration we depict the shapes of $\overset{\circ}{r}(t)$, $\Delta(t)$, and $r(t)$ in Fig. 2.2.1 for a Weibull type linear failure rate ($\alpha = 2$).

When $\Delta(t)$ is a step-like function, the $r(t)$ has a constant slope in each interval. When $\Delta(t)$ is a piecewise linear function, the $r(t)$ has a varying slope in each interval. This corresponds to a <u>maintenance-dependent</u> failure rate parameter sequence $\{\lambda_n\}$.

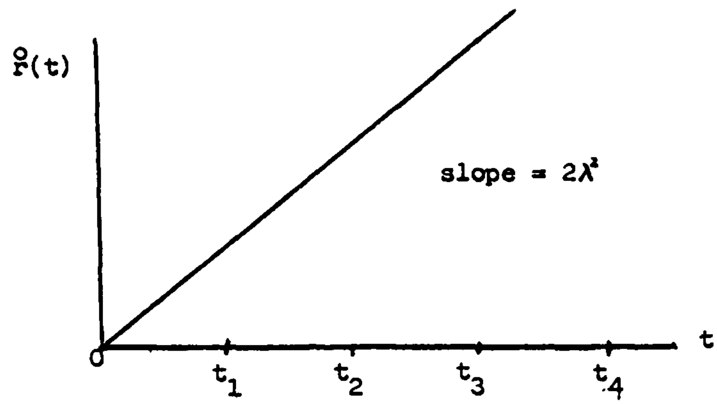


Fig. 2.2.1(a) Regular linear failure rate.

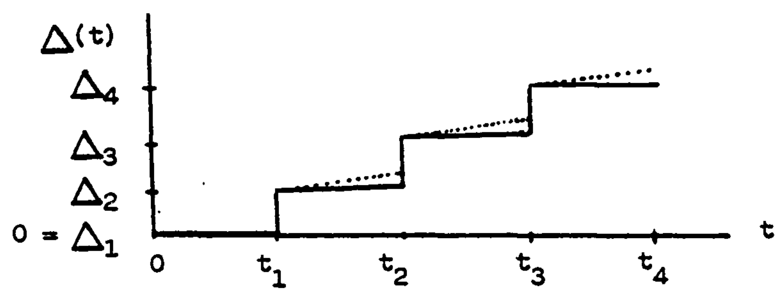


Fig. 2.2.1(b) Failure reduction function.

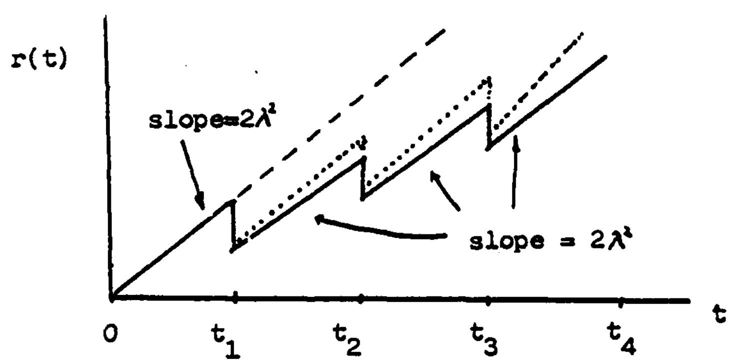


Fig. 2.2.1(c) Linear failure rate with failure reduction.

§ 2.3 FAILURE REDUCTION CRITERIA

For a real, usually large, system, it is more efficient to perform preventive maintenance on a periodic schedule in order to reduce management cost etc. It is therefore reasonable to assume that the operative-cycle times and maintenance-cycle times are fixed (Eq.(2.1.1)),

$$\left.\begin{aligned} u_1 = u_2 = \dots = u_n = \dots &\equiv u \\ d_1 = d_2 = \dots = d_n = \dots &\equiv d \\ t_n = nt_1 \quad , \quad n &= 1,2,3,\dots \end{aligned}\right\} \quad (2.3.1)$$

Furthermore, we may assume that

$$\lambda_1 = \lambda_2 = \dots = \lambda_n = \dots \equiv \lambda \qquad (2.3.2)$$

and

$$g_1 = g_2 = \dots = g_n = \dots \equiv g \in [0,1] \qquad (2.3.3)$$

Two special types of failure reduction are defined in the following :

<u>TYPE 1</u> <u>FIXED FAILURE REDUCTION</u>

After each M-cycle the failure rate is reduced in a way that all jump-downs are the same in each O-M cycle, or more precisely,

$$\left.\begin{aligned} \Delta_1 &= 0 \\ \Delta_2 &= g.r_1(t_1) \equiv \Delta \\ \Delta_3 &= \Delta_2 + \Delta = 2\Delta \\ &\vdots \\ \Delta_n &= \Delta_{n-1} + \Delta = (n-1).\Delta \\ &\vdots \end{aligned}\right\} \quad (2.3.4)$$

where

$$g \in [0,1]$$

Thus,

$$\alpha = 2 \implies \Delta \equiv 2\lambda^2 t_1 g \tag{2.3.5}$$

The failure rate is given by

$$r(t) = \begin{cases} 2\lambda^2 t & , \ t \in [t_0, t_1) \\ 2\lambda^2 t - \Delta & , \ t \in [t_1, t_2) \\ 2\lambda^2 t - 2\Delta & , \ t \in [t_2, t_3) \\ \vdots & \\ 2\lambda^2 t - (n-1)\Delta & , \ t \in [t_{n-1}, t_n) \\ \vdots & \end{cases} \tag{2.3.6}$$

The shape of $r(t)$ is sketched in Fig. 2.3.1. The $r(t)$ given by Eq.(2.3.6) will be called the <u>fixed reductuion piecewisely linear failure rate</u>.

The case when the failure rate parameter λ is not constant is also of interest. We shall assume that λ is actually <u>maintenance-dependent</u>, namely, λ varies in the form of an increasing sequence as in (2.2.17). This is the case when the mean lifetime of the repaired system decreases with the number of preventive maintenance carried out. (From Eq.(A1,4), the mean lifetime is inversely proportional to the λ's.) We shall illustrate this case in Fig. 4.4.8 where the λ's are defined by Eq.(4.4.4).

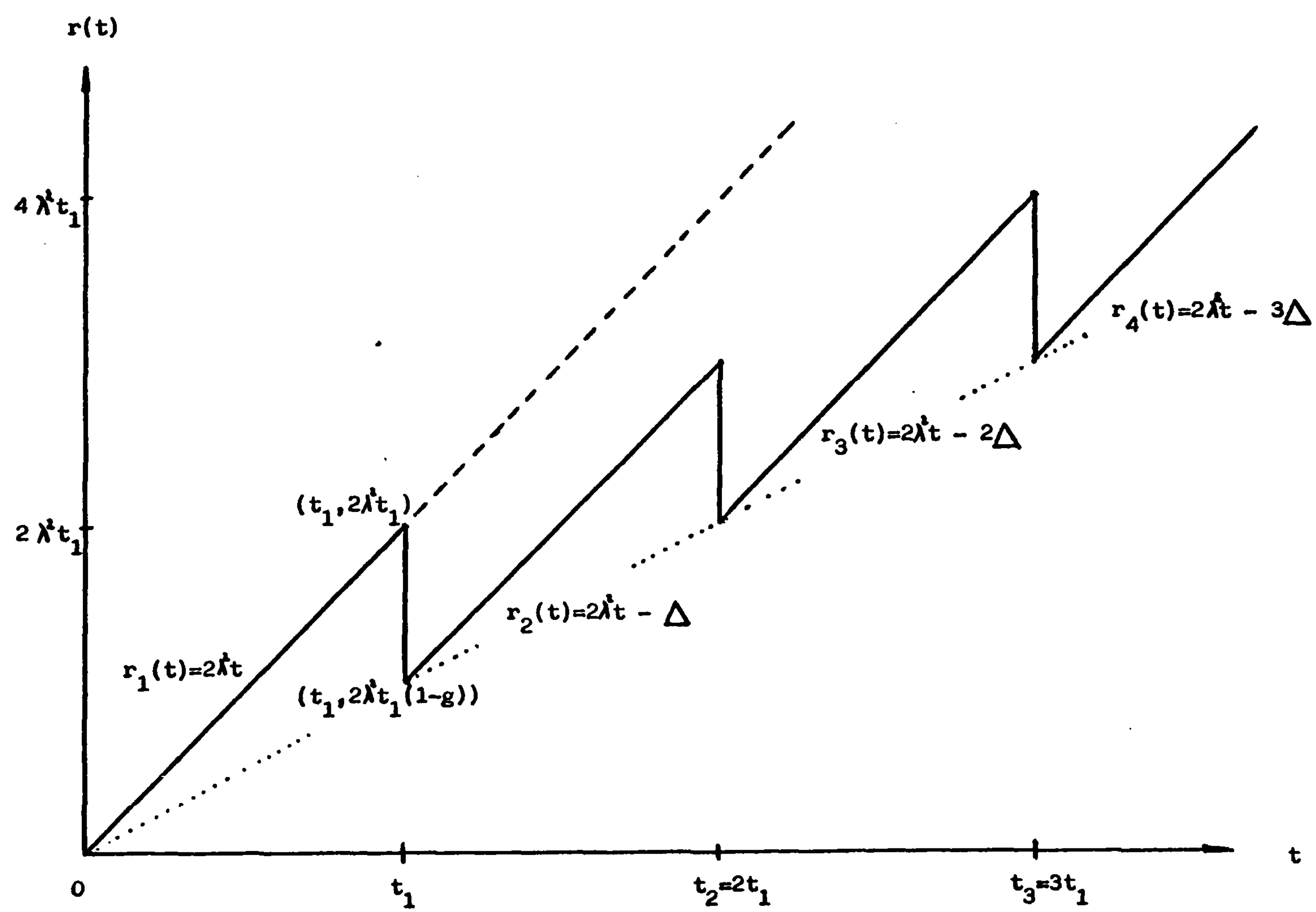


Fig. 2.8.1 Fixed reduction linear failure rate.

TYPE 2 PROPORTIONAL FAILURE REDUCTION

After each M-cycle the failure rate is reduced such that each "jump-down" is proportional to the value of the failure rate at that instant, namely,

$$\left.\begin{aligned}
\Delta_1 &= 0 \\
\Delta_2 &= g \cdot r_1(t_1) \\
\Delta_3 &= g \cdot r_2(t_2) + g \cdot r_1(t_1) \\
&\vdots \\
\Delta_n &= g \cdot r_n(t_n) + \dots + g \cdot r_1(t_1) \\
&\vdots \\
\text{where} \quad & g \in [0,1]
\end{aligned}\right\} \qquad (2.3.7)$$

Thus, for $\alpha = 2$,

$$\left.\begin{aligned}
r_1(t) &= 2\lambda^2 t \\
r_2(t) &= 2\lambda^2 t - 2\lambda^2 g t_1 \\
r_3(t) &= 2\lambda^2 t - 2\lambda^2 g(t_2 + (1-g)t_1) \\
&\vdots
\end{aligned}\right\} \qquad (2.3.8)$$

The shape of $r(t)$ is sketched in Fig. 2.3.2. The $r(t)$ given by Eq.(2.3.8) will be called the proportional reduction piecewisely linear failure rate.

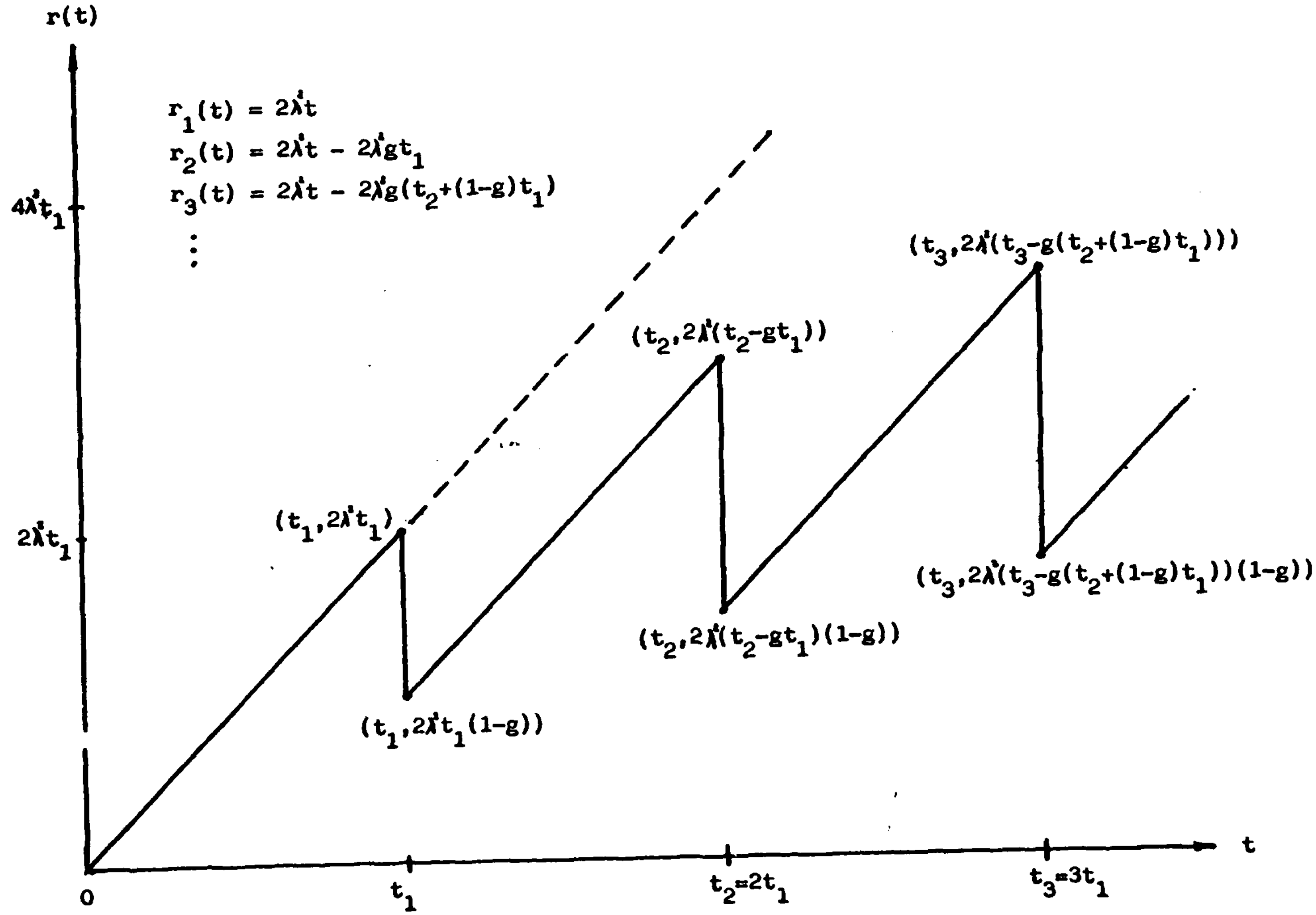


Fig. 2.3.2 Proportional reduction linear failure rate.

§ 2.4 RELIABILITY FUNCTIONS

Recall that the system reliability function $R(t)$ is related to the failure rate function $r(t)$ by Eq.(2.2.6).

Define

$$\overset{o}{r}(t) \equiv \lambda^{\alpha}\alpha t^{\alpha-1} \equiv \text{regular failure rate} \tag{2.4.1}$$

$$\overset{o}{R}(t) \equiv \exp(-\int_0^t r(x)dx) \equiv \text{regular reliability function} \tag{2.4.2}$$

For an age and maintenance dependent failure rate $r(t)$ defined by (2.2.13) - (2.2.21), we define

$$R(t) \equiv \exp(-\int_0^t r(x)\,dx) \tag{2.4.3}$$

and for $t \geqslant t_{n-1}$,

$$R(t) = \exp\left(-\int_0^{t_1} r(x)dx - \int_{t_1}^{t_2} r(x)dx - \dots - \int_{t_{n-1}}^{t} r(x)dx\right) \tag{2.4.4}$$

$$= \left[\prod_{j=1}^{n-1} R_j(t_j)\right] \exp\left(-\int_{t_{n-1}}^{t} r(x)dx\right) \tag{2.4.4'}$$

where

$$R_j(t) \equiv \exp\left(-\int_{t_{j-1}}^{t} r(x)dx\right), \quad t \in [t_{j-1}, t_j), \; j=1,2,\dots \tag{2.4.5}$$

If all 's in (2.2.17) are constant, then

$$r_n(t) = \overset{o}{r}(t) - \Delta_n$$

which, together with Eq.(2.4.5), give

$$R_j(t) = \exp\left(-\int_{t_{j-1}}^{t} r_j(x)dx\right)$$

$$= \exp\left(-\int_{t_{j-1}}^{t} \overset{o}{r}(x)dx\right).\exp\left(\left[t-t_{j-1}\right]\Delta_j\right) \tag{2.4.6}$$

Therefore, Eq.(2.4.4') becomes

$$R(t) = \overset{o}{R}(t).\exp\left[\sum_{j=1}^{n-1}(t_j-t_{j-1})\Delta_j + (t-t_{n-1})\Delta_n\right] \tag{2.4.7}$$

(1) FIXED REDUCTION RELIABILITY FUNCTION

for fixed reduction, (2.3.4) and (2.4.7) gives

$$R(t) = \overset{o}{R}(t).\exp\left\{ \left[\sum_{j=1}^{n-1} (j-1)(t_j - t_{j-1}) + (n-1)(t-t_{n-1}) \right]\Delta \right\} \qquad (2.4.8)$$

For constant operative-cycle and maintenance-cycle times (periodic operative-maintenance schedule),

$$t_n = n.t_1 \qquad (2.4.9)$$

we have,

$$R(t) = \overset{o}{R}(t).\exp\left[(n-1)(t - \tfrac{1}{2}nt_1)\Delta \right] \qquad (2.4.10)$$

$$\alpha = 2 \implies R(t) = \exp(-\lambda^2 t^2 + 2(n-1)(t-\tfrac{1}{2}nt_1)g\,\lambda^2 t_1) \qquad (2.4.11)$$

(2) PROPORTIONAL REDUCTION RELIABILITY FUNCTION

For proportional reduction, with r(t) given by Eq.(2.3.9), we have,

$$R(t) = \begin{cases} \exp(-\lambda^2 t^2) & , \ t \in [0,t_1) \\ \exp(-\lambda^2 t^2 + 2\lambda^2 g t_1 (t-t_1)\ ) & , \ t \in [t_1,t_2) \\ \exp(-\lambda^2 t^2 + 2\lambda^2 g[t_2+(1-g)t_1]t - 2\lambda^2 g(t_1^2+t_2^2-gt_1 t_2)\ ) & , \ t \in [t_2,t_3) \\ \vdots & \end{cases} \qquad (2.4.12)$$

§ 2.5 PROBABILITY DENSITY FUNCTIONS

The probability density function $f(t)$ of the lifetime T is related to the failure rate $r(t)$ and the reliability function $R(t)$ by Eq.(2.2.7). Define

$$\overset{\circ}{f}(t) \equiv \overset{\circ}{r}(t)\overset{\circ}{R}(t) \equiv \text{regular lifetime p.d.f.} \tag{2.5.1}$$

and

$$t \in [t_{n-1}, t_n) \implies f(t) = f_n(t) = r_n(t)R(t) \tag{2.5.2}$$

Thus,

$$f(t) = \begin{cases} f_1(t) \equiv r_1(t)R_1(t) & , \quad t \in [0, t_1) \\ f_2(t) \equiv r_2(t)R_1(t)R_2(t) & , \quad t \in [t_1, t_2) \\ \vdots & \\ f_{n(t)} \equiv r_n(t)R_n(t)\prod_{j=1}^{n-1} R_j(t_j) & , \quad t \in [t_{n-1}, t_n) \\ \vdots & \end{cases} \tag{2.5.3}$$

Note that

$$t \in [0, t_1) \implies f_1(t) = \overset{\circ}{f}(t) \tag{2.5.4}$$

(1) <u>FIXED REDUCTION PROBABILITY DENSITY FUNCTION</u>

For fixed reduction failure rate, Eq.(2.3.6), (2.4.10) and (2.5.2) give

$$f_n(t) = \left(\overset{\circ}{r}(t) - (n-1)\Delta \right) \overset{\circ}{R}(t) \exp\left((n-1)(t - \tfrac{1}{2}nt_1)\Delta \right) \tag{2.5.5}$$

$$= \left(\overset{\circ}{f}(t) - (n-1)\Delta\, \overset{\circ}{R}(t) \right) \exp\left((n-1)(t - \tfrac{1}{2}nt_1)\Delta \right) \tag{2.5.5'}$$

$$\alpha = 2 \implies \Delta \equiv 2\lambda^2 g t_1$$

$$f_n(t) = \left[2\lambda^2 t - (n-1)\Delta \right] \exp\left(-\lambda^2 t^2 + (n-1)(t - \tfrac{1}{2}nt_1)\Delta \right) \tag{2.5.6}$$

and the shape of $f(t)$ is sketched in Fig.2.5.1.

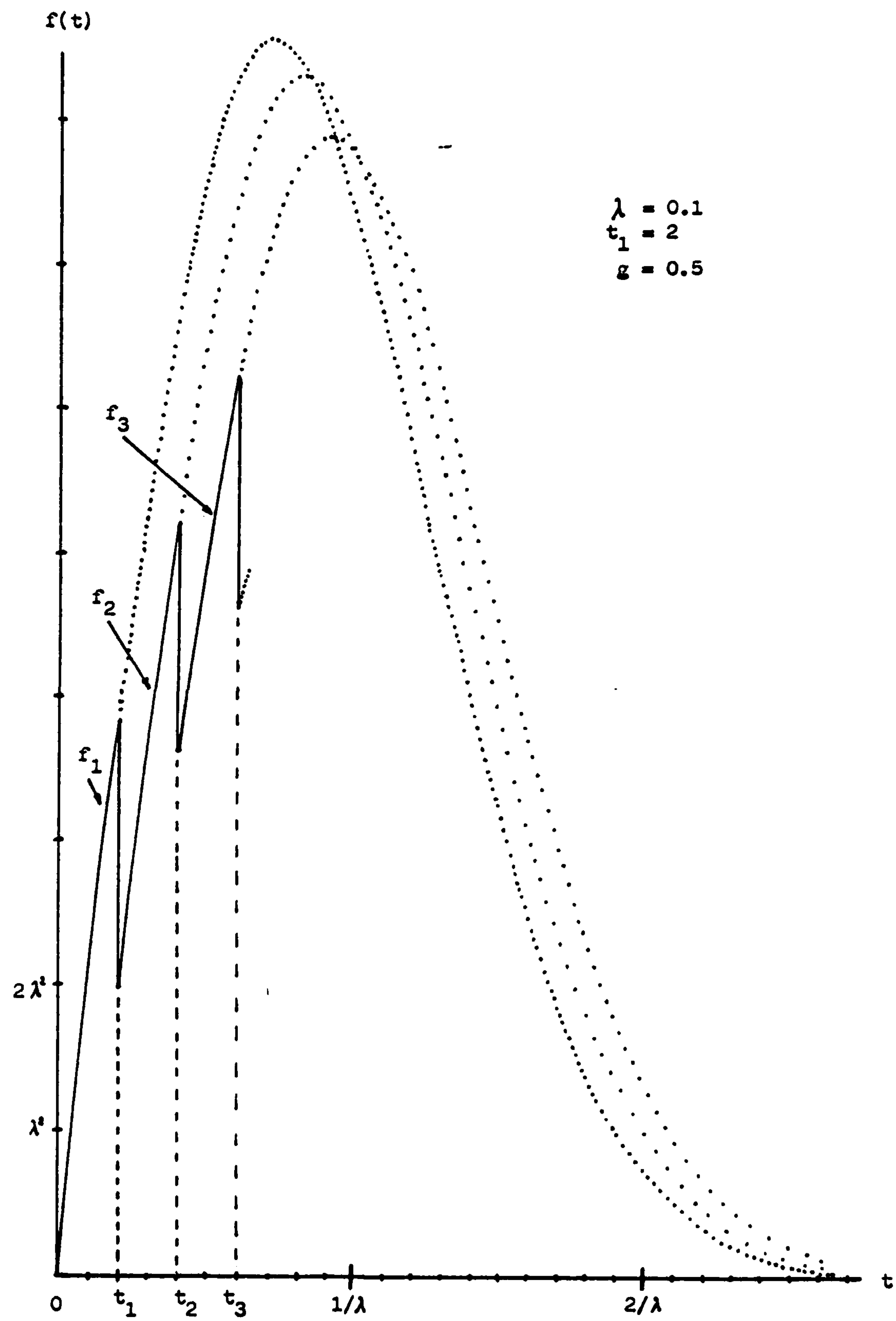


Fig. 2.5.1 Lifetime probability density function with fixed reduction.

(2) PROPORTIONAL REDUCTION PROBABILITY DENSITY FUNCTION

For proportional reduction failure rate, we have for $\alpha = 2$,

$$f(t) = \begin{cases} 2\lambda^2 t.\exp(-\lambda^2 t^2) & , t \in [0, t_1) \\ 2\lambda^2 (t-gt_1).\exp\left|- \quad t^2 + 2 \quad gt_1(t-t_1)\right| & , t \in [t_1, t_2) \\ 2\lambda^2\left[t-g(t_2+(1-g)t_1)\right]\exp\left\{-\lambda^2\left[t^2-2g(t_2+(1-g)t_1)t + 2g(t_1^2+t_2^2-gt_1t_2)\right]\right\} & , t \in [t_2, t_3) \\ \vdots & \end{cases} \tag{2.5.7}$$

The shape of $f(t)$ is sketched in Fig. 2.5.2.

* * * * *

Note that $f_1(t)$ and $f_2(t)$ are the same for both fixed and proportional reductions for a given failure reduction factor g (see Fig. 2.5.1 and 2.5.2). Suppose there is only <u>one</u> maintenance-cycle and a large operative-cycle time. In this case both fixed and proportional reductions are equivalent. The curves of $\mathring{f}(t)$ and $f(t)$ (for $\alpha = 2$) are illustrated in Fig. 2.5.3, and

$$\mathring{f}(t) = 2\lambda^2 t \exp(-\lambda^2 t^2) , \quad t \geq 0 \tag{2.5.8}$$

$$f(t) = \begin{cases} 2\lambda^2 t \exp(-\lambda^2 t^2) & , t \in [0, t_1) \\ (2\lambda^2 t - \Delta)\exp\left\{-\lambda^2 t^2 + (t-t_1)\Delta\right\} & , t \geq t_1 \end{cases} \quad ; \Delta \equiv 2g\lambda^2 t_1 \tag{2.5.9}$$

Let $t^{¢}$ be the point that both curves intersect, that is,

$$\mathring{f}(t^{¢}) - f(t^{¢}) = 0 \tag{2.5.10}$$

or,

$$\exp(-\left[t^{¢}-t_1\right]\Delta) = 1 - \frac{\Delta}{2\lambda^2 t^{¢}}$$

Expanding the exponential, we have approximately,

$$t^{¢} \approx \frac{1}{2} t_1 \left[1 + \sqrt{(1 + \frac{2}{\lambda^2 t_1^2})}\right] \tag{2.5.11}$$

$$t^{¢} - t_1 \sim \frac{1}{2\lambda^2 t_1} \quad \text{for large } t_1 \tag{2.5.12}$$

$$= O(\frac{1}{t_1}) \tag{2.5.12'}$$

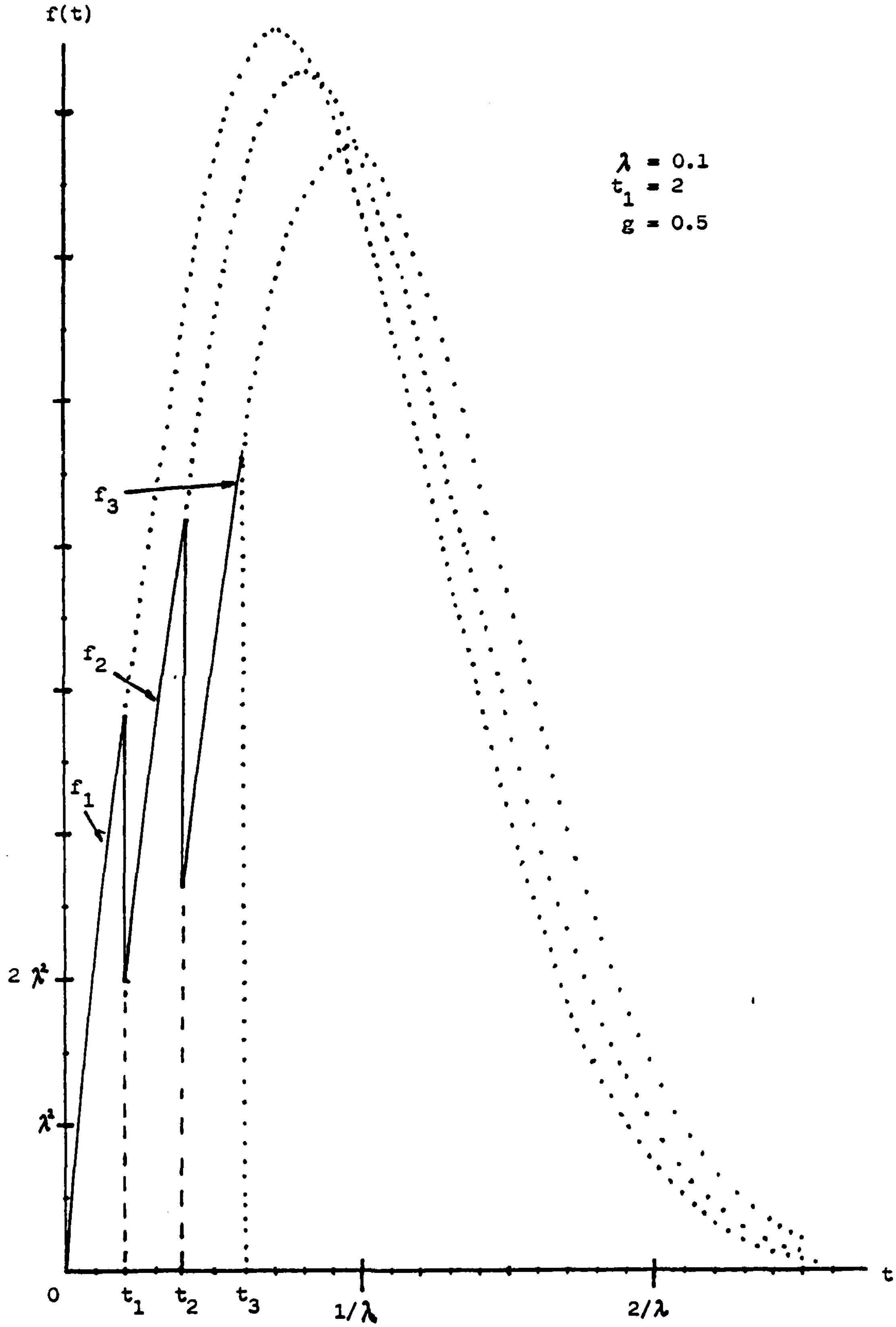


Fig. 2.5.2 Lifetime probability density function with proportional reduction.

$$t_1 \to \infty \quad \Longrightarrow \quad t^{\not c} \downarrow t_1 \quad . \tag{2.5.13}$$

And

$$\overset{o}{f}(t) > f(t) \quad \text{for} \quad t < t^{\not c} \tag{2.5.14}$$

$$f(t) > \overset{o}{f}(t) \quad \text{for} \quad t > t^{\not c} \tag{2.5.15}$$

We conclude that for large operative-cycle time and a single maintenance operation, the lifetime probability density function with failure reduction is always greater than the regular lifetime p.d.f. whenever $t > t_1$, namely,

$$t > t^{\not c} \approx t_1 \quad \Longrightarrow \quad f(t) > \overset{o}{f}(t) \tag{2.5.16}$$

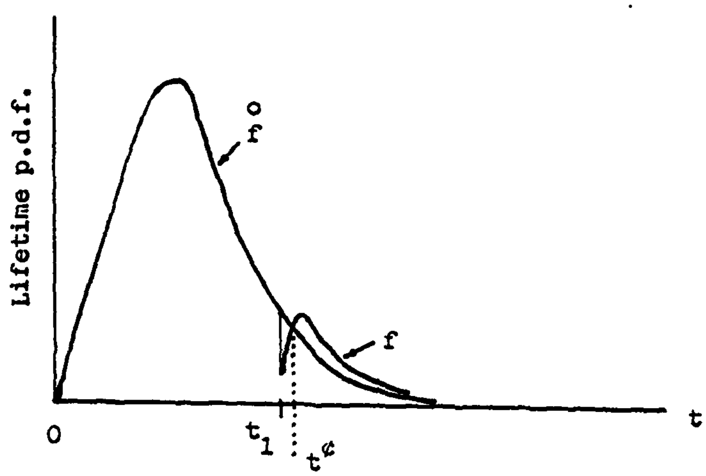


Fig. 2.5.3 Asymptotic behavior of lifetime p.d.f. for a single maintenance.

§ 2.6 OPERATIVE TIME AND MAINTENANCE TIME DISTRIBUTIONS

We define an integer random variable N_t to be the number of O-M cycles before failure. It is obvious from Fig. 2.1.2 that

$$N_t = n \quad \Longleftrightarrow \quad T = t \in [t_{n-1}, t_n) \qquad (2.6.1)$$

for $n = 1,2,3,\ldots$, and $t_0=0$.

Note that for a renewal process (Parzen (1962)) the expectation of N_t is called the renewal function. We shall use the expected value of N_t, namely, $E[N_t]$, to compute the approximate mean uptime and the approximate mean lifetime for a periodic operative-maintenance schedule (Eq.(2.6.22) and (2.6.23)).

Since the system may fail during the n^{th} O-cycle (i.e. in $[t_{n-1}, \tau_n)$) or during the n^{th} M-cycle (i.e. in $[\tau_n, t_n)$), we further define the following two randowm variables $T_{u|n}$ and $T_{d|n}$ related to the system lifetime T :

For $n = 1,2,3,\ldots$

$T_{u|n} \equiv$ operative time before failure in the n^{th} O-cycle (the n^{th} O-cycle operative time)

$T_{d|n} \equiv$ maintenance time before failure in the n^{th} M-cycle (the n^{th} M-cycle maintenance time)

such that

$$T = \begin{cases} t_{n-1} + T_{u|n}, & \text{if } (O) \rightarrow (F) \text{ when } N_t=n \\ \\ \tau_n + T_{d|n}, & \text{if } (M) \rightarrow (F) \text{ when } N_t=n \end{cases} \qquad (2.6.3)$$

It is evident that

$$0 \leq T_{u|n} \leq u_n \tag{2.6.4}$$

and

$$0 \leq T_{d|n} \leq d_n \tag{2.6.5}$$

Furthermore,

$$0 \leq T_{u|n} < u_n \quad \Longleftrightarrow \quad T_{d|n} = 0 \tag{2.6.6}$$

$$0 < T_{d|n} \leq d_n \quad \Longleftrightarrow \quad T_{u|n} = u_n \tag{2.6.7}$$

The relationship of T, $T_{u|n}$ and $T_{d|n}$ is illustrated in Fig. 2.6.1.

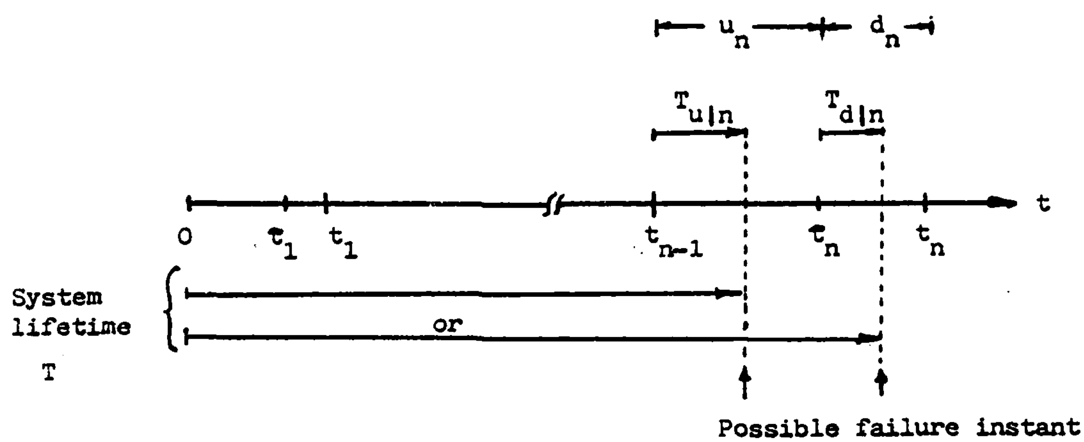


Fig. 2.6.1 Lifetimes relationship before failure.

Next, we shall compute the probability density functions of $T_{u|n}$ and $T_{d|n}$, denoted respectively by

$$f_{T_{u|n}} \quad \text{and} \quad f_{T_{d|n}}$$

Suppose $N_t = n$ and the p.d.f. of T assuues the shape shown in Fig. 2.6.2 in which the $f_n(t)$ is defined by Eq.(2.5.2)-(2.5.3).

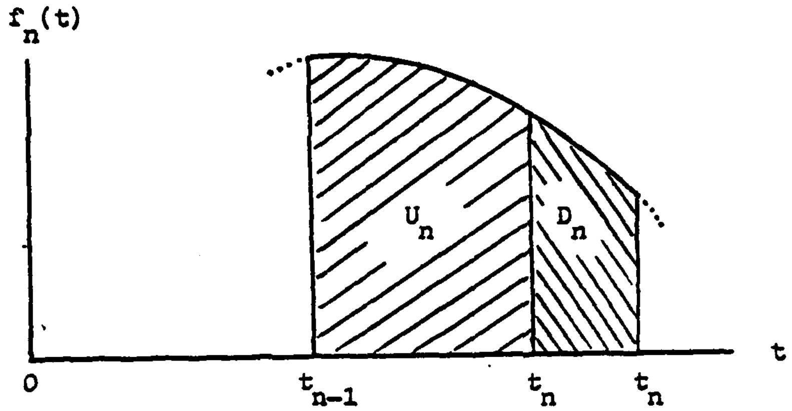


Fig. 2.6.2 Lifetime probability density function in the n^{th} O-M cycle.

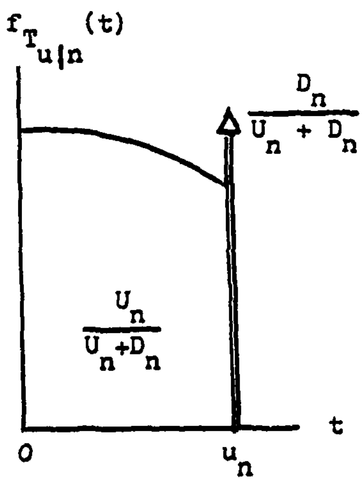


Fig. 2.6.3 P.d.f. of $T_{u|n}$.

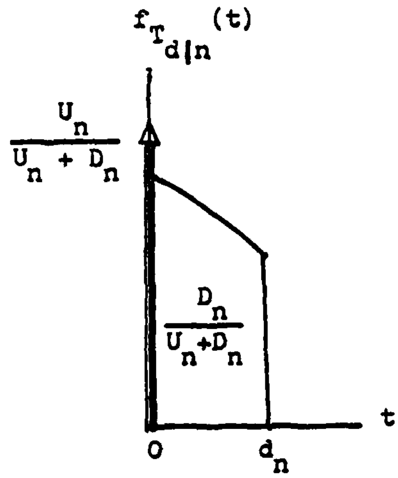


Fig. 2.6.4 P.d.f. of $T_{d|n}$.

Let U_n and D_n be the shaded areas in Fig.2.6.2, that is,

$$U_n = \int_{t_{n-1}}^{\tau_n} f_n(x)dx \tag{2.6.8}$$

and

$$D_n = \int_{\tau_n}^{t_n} f_n(x)dx \tag{2.6.9}$$

Obviously,

$$\begin{aligned} P\{N_t = n\} &= P\{t_{n-1} \leq T < t_n\} \\ &= \int_{t_{n-1}}^{t_n} f_n(x)dx \\ &= U_n + D_n \end{aligned} \tag{2.6.10}$$

The p.d.f.'s $f_{T_{u|n}}$ and $f_{T_{d|n}}$ have the same shape as that of the p.d.f. of T in $[t_{n-1}, \tau_n)$ and $[\tau_n, t_n)$ respectively. Both are depicted in Fig.(2.6.3) and (2.6.4).

From Eq.(2.6.6) and (2.6.7), we have

$$P\{0 \leq T_{u|n} < u_n\} = P\{T_{d|n} = 0\} = \frac{U_n}{U_n + D_n} \tag{2.6.11}$$

$$P\{0 < T_{d|n} \leq d_n\} = P\{T_{u|n} = u_n\} = \frac{D_n}{U_n + D_n} \tag{2.6.12}$$

Then,

$$f_{T_{u|n}}(t) = \begin{cases} \dfrac{f_n(t_{n-1}+t)}{U_n + D_n} , & 0 \leq t < u_n \\[2ex] \dfrac{D_n}{U_n + D_n} , & t = u_n \\[2ex] 0 , & \text{otherwise} \end{cases} \tag{2.6.13}$$

$$f_{T_{d|n}}(t) = \begin{cases} \dfrac{U_n}{U_n + D_n} , & t = 0 \\[2ex] \dfrac{f_n(\tau_n+t)}{U_n + D_n} , & 0 < t \leq d_n \\[2ex] 0 , & \text{otherwise} \end{cases} \tag{2.6.14}$$

Furthermore, from Eq.(2.5.3),

$$\begin{aligned} U_n &= \int_{t_{n-1}}^{\tau_n} f_n(x)\,dx \\ &= \int_{t_{n-1}}^{\tau_n} r_n(x) R_n(x) \prod_{j=1}^{n-1} R_j(t_j)\,dx \\ &= \left[\prod_{j=1}^{n-1} R_j(t_j) \right] \left[1 - R_n(\tau_n) \right] \end{aligned} \tag{2.6.15}$$

$$\begin{aligned} D_n &= \int_{\tau_n}^{t_n} f_n(x)\,dx \\ &= \int_{\tau_n}^{t_n} r_n(x) R_n(x) \prod_{j=1}^{n-1} R_j(t_j)\,dx \\ &= \left[\prod_{j=1}^{n-1} R_j(t_j) \right] \left[R_n(\tau_n) - R_n(t_n) \right] \end{aligned} \tag{2.6.16}$$

Eq. (2.6.10) and (2.6.16) give

$$P\left\{N_t = n\right\} = U_n + D_n$$

$$= \left[\prod_{j=1}^{n-1} R_j(t_j)\right]\left[\ 1 - R_n(t_n)\ \right] \tag{2.6.17}$$

In particular,

$$n = 1 \implies P\left\{N_t = 1\right\} = 1 - R_1(t_1) \tag{2.6.18}$$

The average number of O-M cycles is given by

$$E\ [\ N_t\ ] \equiv \sum_{n=1}^{\infty} n.P\left\{N_t = n\right\}$$

$$\begin{aligned} &= \left[1 - R_1(t_1)\right] + 2\left[R_1(t_1) - R_1(t_1)R_2(t_2)\right] \\ &\quad + 3\left[R_1(t_1)R_2(t_2) - R_1(t_1)R_2(t_2)R_3(t_3)\right] \\ &\quad + \ldots \\ &\quad + n\left[R_1(t_1)\ldots R_{n-1}(t_{n-1}) - R_1(t_1)\ldots R_{n-1}(t_{n-1})R_n(t_n)\right] \\ &\quad + \ldots \end{aligned}$$

$$= 1 + \sum_{n=1}^{\infty}\left[\prod_{j=1}^{n} R_j(t_j)\right] \tag{2.6.19}$$

Or,

$$E\ [\ N_t\ ] - 1 = \sum_{n=1}^{\infty} \exp\left\{-\sum_{j=1}^{n}\int_{t_{j-1}}^{t_j} r_j(x)dx\right\} \tag{2.6.19'}$$

Similarly, the k^{th} moment of N_t is given by

$$E\ (\ N_t^{\,k}\ ) \equiv \sum_{n=1}^{\infty} n^k.P\left\{N_t = n\right\}$$

$$= \sum_{n=1}^{\infty} n^k.\left[\prod_{j=1}^{n-1} R_j(t_j)\right]\left[\ 1 - R_n(t_n)\ \right] \tag{2.6.20}$$

Note that if there is no maintenance operation, or the operative-cycle time becomes infinite, then

$$t_1 \rightarrow \infty \quad \Longrightarrow \quad R_1(t_1) \rightarrow 0$$

and Eq.(2.6.19) becomes

$$\lim_{t_1 \rightarrow \infty} E[N_t] = 1 \qquad (2.6.21)$$

That is, there is only one O-cycle. (See Table 4.4.1 and Fig. 4.4.3 for a numerical illustration).

We may use the mean O-M cycles $E[N_t]$ to compute the approximate mean uptime and the approximate mean lifetime for a periodic operative-maintenance schedule ((2.1.1)).

Define

$$\text{Approximate mean uptime} \equiv u.(E[N_t] - \tfrac{1}{2}) \qquad (2.6.22)$$

$$\text{Approximate mean lifetime} \equiv (u + d).(E[N_t] - \tfrac{1}{2}) \qquad (2.6.23)$$

We shall see that Eq.(2.6.22) and (2.6.23) give consistent results as $u \rightarrow \infty$ (See Table 4.4.1 and Fig. 4.4.4–4.4.5).

§ 2.7 REPAIR TIME DISTRIBUTION

When ths system fails, it is in the (F) stste for a complete repair, replacomont or ronewal. The time spent is a random variable called the <u>repair time</u> T_F with distribution function

$$\mathcal{P}(t)$$

and probability density function

$$p(t)$$

For simplicity, T_F is assumed to <u>statistically independent</u> of the system lifetime T, and T_F is expenentially distributed,

$$p(t) = \mu \exp(-\mu t) \quad , \quad t \geq 0 \tag{2.7.1}$$

$$\mathcal{P}(t) = 1 - \exp(-\mu t) \quad , \quad t \geq 0 \tag{2.7.2}$$

We shall see that other distributions may be used and the independence assumption may be dropped, but we have to handle double integrals, instead of single integrals in evaluating the probability of stochastic availability (see § 3.2).

CHAPTER 3

STOCHASTIC AVAILABILITY

§ 3.1 DEFINITIONS OF STOCHASTIC AVAILABILITIES

We define the following random variables with reference to Fig. 2.1.2 and 2.6.1 :

T_{up} $\equiv$ total uptime before failure when the system is operative

T_{down} $\equiv$ total downtime during which the system is not operative

$$T_{up|n} \equiv \sum_{j=1}^{n-1} u_j + T_{u|n}$$

$$n = 1,2,3,\ldots$$

$$T_{down|n} \equiv \sum_{j=1}^{n-1} d_j + T_{d|n} + T_F$$

where the summations are defined to be zero for $n = 1$.

The following stochastic availabilities (A) are defined :

(1) Stochastic Cycle Availability (Stochastic Availability for a complete Renewal Cycle) A_c

$$A_c \equiv \frac{T_{up}}{T_{up} + T_{down}} \tag{3.1.1}$$

(2) <u>Stochastic Availability for the n^{th} O-M Cycle</u> $A_{N_t=n}$ (A_n)

$$A_{N_t=n} \equiv A_n \equiv \frac{T_{up|n}}{T_{up|n} + T_{down|n}} \tag{3.1.2}$$

$$= \frac{\sum_{j=1}^{n-1} u_j + T_{u|n}}{\sum_{j=1}^{n-1} u_j + T_{u|n} + \sum_{j=1}^{n-1} d_j + T_{d|n} + T_F} \tag{3.1.2'}$$

(3) <u>Stochastic Availability for the n^{th} O-cycle</u> A_{O_n}

$$A_{O_n} \equiv \frac{\sum_{j=1}^{n-1} u_j + T_{u|n}}{\sum_{j=1}^{n-1} u_j + T_{u|n} + \sum_{j=1}^{n-1} d_j + T_F} \tag{3.1.3}$$

$$= A_{N_t=n} \Big|_{T_{d|n}=0} \tag{3.1.3'}$$

(4) <u>Stochastic Availability for the n^{th} M-cycle</u> A_{M_n}

$$A_{M_n} \equiv \frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + \sum_{j=1}^{n-1} d_j + T_{d|n} + T_F} \tag{3.1.4}$$

$$= A_{N_t=n} \Big|_{T_{u|n}=u_n} \tag{3.1.4'}$$

(5) Stochastic Availability at a Finite Time in the n^{th} O-M Cycle A_n^t
(Finite Time Stochastic Availability)

$$A_n^t \equiv \begin{cases} \dfrac{\sum_{j=1}^{n-1} u_j + \min(\hat{t}, T_{u|n})}{\sum_{j=1}^{n-1} u_j + \min(\hat{t}, T_{u|n}) + \sum_{j=1}^{n-1} d_j + T_F}, & t = t_{n-1} + \hat{t} < \tau_n \\[2ex] \dfrac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + \sum_{j=1}^{n-1} d_j + \min(\check{t}, T_{d|n}) + T_F}, & t = \tau_n + \check{t} < t_n \end{cases} \tag{3.1.5}$$

Hence,

$$A_{O_n} = A_n^{t=\tau_n} \tag{3.1.5'}$$

$$A_{M_n} = A_n^{t=t_n} \tag{3.1.5''}$$

(6) Stochastic Availability for an Age Replacement Time $A_{AR}^{t_R}$

$$A_{AR}^{t_R} \equiv \frac{\text{uptime before failure in } [0,t_R)}{\text{uptime before failure in } [0,t_R) + \text{Total downtime}} \tag{3.1.6}$$

where t_R is the age replacement time. See § 4.2 for more detail.

* * * * * * * * * * *

We note that the stochastic availabilities defined above are in the form

$$A = \frac{\text{uptime}}{\text{uptime} + \text{downtime}} \tag{3.1.7}$$

where the uptime means that the system is operative and the downtime means that it is not operative such as in the maintenance or failure state. Since the uptime and/or the downtime are in general, nonnegative random variables, the stochastic availability is therefore random and less than unity.

§ 3.2 PROBABILITY DISTRIBUTIONS OF STOCHASTIC AVAILIABILITY

We have defined various stochastic availabilities A in the previous section. We shall derive the probability of the form

$$P\left\{ A \geqslant 1-\varepsilon \right\} \quad , \quad 0 < \varepsilon < 1$$

in this section and in § 4.2. We shall begin with the stochastic cycle availability.

By the total probability theorem,

$$P\left\{ A_c \geqslant 1-\varepsilon \right\}$$

$$= \sum_{n=1}^{\infty} P\left\{ A_c \geqslant 1-\varepsilon \mid N_t = n \right\} . P\left\{ N_t = n \right\}$$

$$= \sum_{n=1}^{\infty} P\left\{ A_n \geqslant 1-\varepsilon \right\} P\left\{ N_t = n \right\} \tag{3.2.1}$$

$$= \sum_{n=1}^{\infty} \left(\begin{array}{l} P\left\{ A_n \geqslant 1-\varepsilon \mid T_{d|n} = 0 \right\} P\left\{ T_{d|n} = 0 \right\} \\ + \\ P\left\{ A_n \geqslant 1-\varepsilon \mid T_{u|n} = u_n \right\} P\left\{ T_{u|n} = u_n \right\} \end{array} \right) P\left\{ N_t = n \right\}$$

$$= \sum_{n=1}^{\infty} \left(\begin{array}{l} P\left\{ A_{O_n} \geqslant 1-\varepsilon \right\} P\left\{ T_{d|n} = 0 \right\} \\ + \\ P\left\{ A_{M_n} \geqslant 1-\varepsilon \right\} P\left\{ T_{u|n} = u_n \right\} \end{array} \right) P\left\{ N_t = n \right\} \tag{3.2.2}$$

Consider $\qquad A_{O_n} \geqslant 1 - \varepsilon$

$$\Rightarrow \qquad \frac{\sum_{j=1}^{n-1} u_j + T_{u|n}}{\sum_{j=1}^{n-1} u_j + T_{u|n} + \sum_{j=1}^{n-1} d_j + T_F} \geqslant 1 - \varepsilon$$

$$\Rightarrow \qquad T_F \leqslant \frac{\varepsilon}{1-\varepsilon} T_{u|n} + \frac{\varepsilon}{1-\varepsilon} \sum_{j=1}^{n-1} u_j - \sum_{j=1}^{n-1} d_j \tag{3.2.3}$$

The domain of definition of $T_{u|n}$ and T_F is shown in Fig. 3.2.1.

Let $f_{T_{u|n},T_F}(\cdot,\cdot)$ be the joint probability density function of $T_{u|n}$ and T_F.

By the independence assumption (§ 2.7), we have

$$f_{T_{u|n},T_F}(x,y) = f_{T_{u|n}}(x)p(y) \tag{3.2.4}$$

Define

$$0^{(+)} \equiv \max\left\{ 0\,,\ -\sum_{j=1}^{n-1} u_j + \frac{1-\varepsilon}{\varepsilon}\sum_{j=1}^{n-1} d_j \right\} \tag{3.2.5}$$

$$u_n^{(-)} \equiv \max\left\{ u_n,\ -\sum_{j=1}^{n-1} u_j + \frac{1-\varepsilon}{\varepsilon}\sum_{j=1}^{n-1} d_j \right\} \tag{3.2.6}$$

And

$$\begin{aligned} t_{n-1}^{(+)} &\equiv t_{n-1} + 0^{(+)} \\ &= \max\left\{ t_{n-1}\,,\ \frac{1}{\varepsilon}\sum_{j=1}^{n-1} d_j \right\} \end{aligned} \tag{3.2.7}$$

$$\begin{aligned} \tau_n^{(-)} &\equiv t_{n-1} + u_n^{(-)} \\ &= \max\left\{ \tau_n\,,\ \frac{1}{\varepsilon}\sum_{j=1}^{n-1} d_j \right\} \end{aligned} \tag{3.2.8}$$

Then

$$P\left\{ A_{0_n} \geq 1-\varepsilon \right\}$$

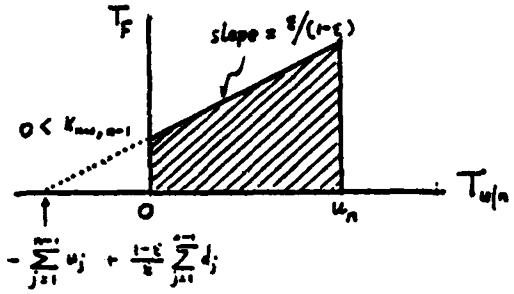
$$= P\left\{ T_F \leq \frac{\varepsilon}{1-\varepsilon} T_{u|n} + \frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n-1} u_j - \sum_{j=1}^{n-1} d_j \right\}$$

$$= \iint_{\Omega} f_{T_{u|n},T_F}(x,y)\, dxdy$$

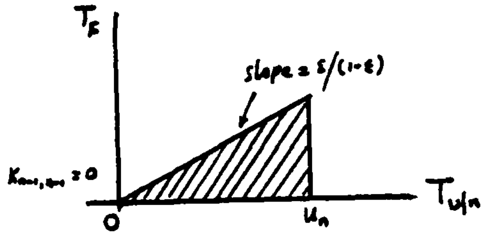
$$= \int_{x=0^{(+)}}^{u_n^{(-)}} f_{T_{u|n}}(x) \int_{y=0}^{\frac{\varepsilon}{1-\varepsilon}x + \frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n-1} u_j - \sum_{j=1}^{n-1} d_j} p(y)\, dy\, dx$$

$$= \int_{0^{(+)}}^{u_n^{(-)}} f_{T_{u|n}}(x)\ \mathcal{P}\left(\frac{\varepsilon}{1-\varepsilon}(x + t_{n-1}) - \frac{1}{1-\varepsilon}\sum_{j=1}^{n-1} d_j \right) dx$$

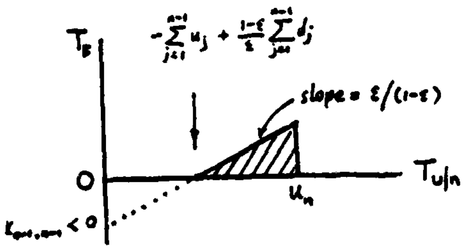
$$T_F \leqslant \frac{\varepsilon}{1-\varepsilon} T_{u|n} + \underbrace{\frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n-1} u_j - \frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n-1} d_j}_{k_{n-1,n-1}}$$



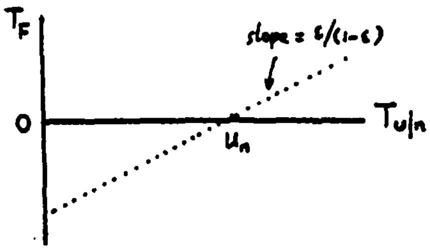
$$\frac{\sum_{j=1}^{n-1} u_j}{\sum_{j=1}^{n-1} u_j + \sum_{j=1}^{n-1} d_j} > 1-\varepsilon$$



$$\frac{\sum_{j=1}^{n-1} u_j}{\sum_{j=1}^{n-1} u_j + \sum_{j=1}^{n-1} d_j} = 1-\varepsilon$$



$$\frac{\sum_{j=1}^{n-1} u_j}{\sum_{j=1}^{n-1} u_j + \sum_{j=1}^{n-1} d_j} < 1-\varepsilon < \frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + \sum_{j=1}^{n-1} d_j}$$



$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + \sum_{j=1}^{n-1} d_j} \leqslant 1-\varepsilon$$

Fig. 3.2.1 Domain of definition of $T_{u|n}$ and T_F .

Let $z = x + t_{n-1}$ and from Eq.(2.6.13), we have

$$P\left\{ A_{O_n} \geq 1-\varepsilon \right\} = \int_{t_n^{(+)}}^{\tau_n^{(-)}} \frac{1}{U_n+D_n} f_n(z)\, \mathcal{P}\left(\frac{\varepsilon}{1-\varepsilon}z - \frac{1}{1-\varepsilon}\sum_{j=1}^{n-1} d_j\right) dz \tag{3.2.9}$$

$$\equiv \frac{\mathcal{U}_n}{U_n + D_n} \tag{3.2.10}$$

Next, consider

$$A_{M_n} \geq 1 - \varepsilon$$

$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + \sum_{j=1}^{n-1} d_j + T_{u|n} + T_F} \geq 1 - \varepsilon$$

$$T_F + T_{d|n} \leq \frac{\varepsilon}{1-\varepsilon} \sum_{j=1}^{n} u_j - \sum_{j=1}^{n-1} d_j \tag{3.2.11}$$

The domain of definition of $T_{d|n}$ and T_F is shown in Fig. 3.2.2.

Let $f_{T_{d|n},T_F}(\cdot,\cdot)$ be the joint probability density function of $T_{d|n}$ and T_F and by the independence assumption (§ 2.7),

$$f_{T_{d|n},T_F}(x,y) = f_{T_{d|n}}(x)p(y) \tag{3.2.12}$$

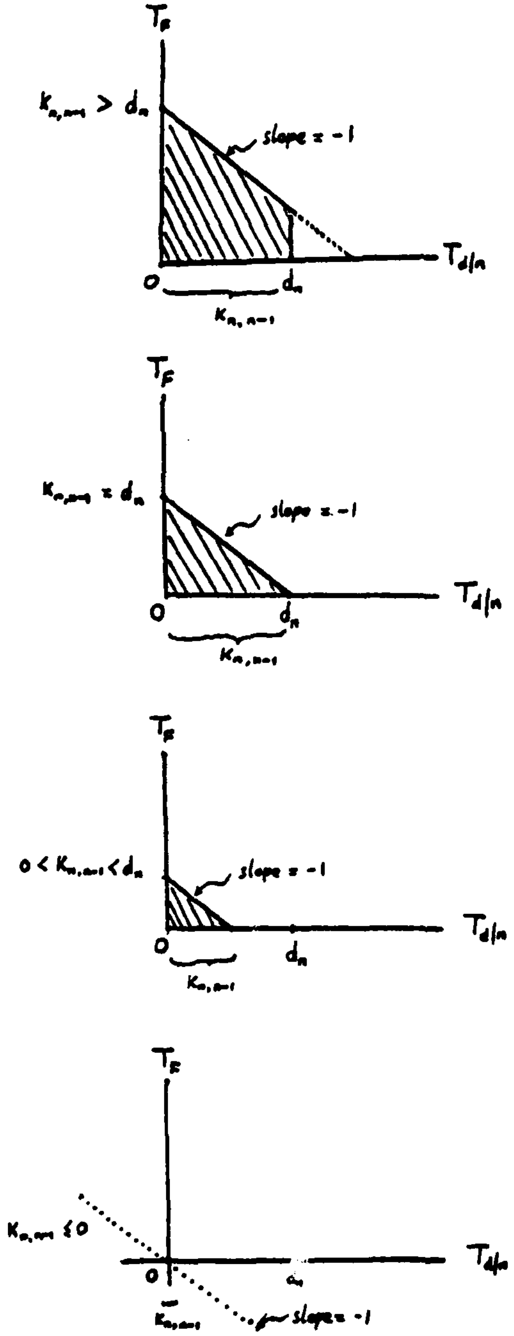
Let

$$d_n^{(-)} \equiv \max\left\{ 0 ,\ \min\left\{ d_n , \frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n} u_j - \sum_{j=1}^{n-1} d_j \right\} \right\} \tag{3.2.13}$$

$$t_n^{(-)} \equiv \tau_n + d_n^{(-)}$$

$$= \max\left\{ \min\left\{ t_n,\ t_n + \frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n} u_j - \sum_{j=1}^{n} d_j \right\},\ \tau_n \right\} \tag{3.2.14}$$

$$T_F + T_{d|n} \lessgtr \underbrace{\frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n-1} u_j - \sum_{j=1}^{n-1} d_j}_{k_{n,n-1}}$$



$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + \sum_{j=1}^{n} d_j} > 1-\varepsilon$$

$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + \sum_{j=1}^{n} d_j} = 1-\varepsilon$$

$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + \sum_{j=1}^{n} d_j} < 1-\varepsilon$$

$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + \sum_{j=1}^{n-1} d_j} < 1-\varepsilon$$

Fig. 3.2.2 Domain of definition of $T_{d|n}$ and T_F .

Then,

$$P\left\{ A_{M_n} \geq 1 - \varepsilon \right\}$$

$$= P\left\{ T_F + T_{d|n} \leq \frac{\varepsilon}{1-\varepsilon} \sum_{j=1}^{n} u_j - \sum_{j=1}^{n-1} d_j \right\}$$

$$= \iint_{D} f_{T_{d|n}, T_F}(x,y)\, dxdy$$

$$= \int_{x=0}^{d_n^{(-)}} f_{T_{d|n}}(x) \int_{y=0}^{-x+\frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n} u_j - \sum_{j=1}^{n-1} d_j} p(y)\, dy\, dx$$

$$= \int_{0}^{d_n^{(-)}} f_{T_{d|n}}(x)\, \mathcal{P}\left(-x + \frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n} u_j - \sum_{j=1}^{n-1} d_j \right) dx$$

$$= \int_{0}^{d_n^{(-)}} f_{T_{d|n}}(x)\, \mathcal{P}\left(-(x+t_n) + \frac{1}{1-\varepsilon}\sum_{j=1}^{n} u_j \right) dx$$

Let $z = x + \tau_n$ and from Eq.(2.6.14), we have

$$P\left\{ A_{M_n} \geq 1 - \varepsilon \right\} = \int_{\tau_n}^{t_n^{(-)}} \frac{1}{U_n + D_n} f_n(z)\, \mathcal{P}\left(- z + \frac{1}{1-\varepsilon} \sum_{j=1}^{n} u_j \right) dz \tag{3.2.15}$$

$$\equiv \frac{\mathcal{D}_n}{U_n + D_n} \tag{3.2.16}$$

From Eq.(3.2.1), (3.2.2), (3.2.10), (3.2.16), (2.6.11) and (2.6.12), we have

$$P\left\{ A_n \geq 1 - \varepsilon \right\} = \frac{U_n \mathcal{U}_n + D_n \mathcal{D}_n}{(U_n + D_n)^2} \tag{3.2.17}$$

Eq.(3.2.17) and (2.6.10) give the probability distribution of the stochastic cycle availability :

$$P\left\{ A_c \geqslant 1-\varepsilon \right\} = \sum_{n=1}^{\infty} \frac{U_n \mathcal{U}_n + D_n \mathcal{D}_n}{U_n + D_n} \tag{3.2.18}$$

The finite time stochastic availability can be computed similarly by the following changes (see Eq.(3.1.5') and (3.1.5")) :

$$\begin{cases} u_n \mapsto \min(\hat{t},u_n) & , \quad 0<\hat{t} \equiv t - t_{n-1} \leqslant u_n \\ \tau_n \mapsto \min(t,\tau_n) \end{cases} \tag{3.2.19}$$

$$\begin{cases} d_n \mapsto \min(\check{t},d_n) & , \quad 0<\check{t} \equiv t - \tau_n \leqslant d_n \\ t_n \mapsto \min(t,t_n) \end{cases} \tag{3.2.20}$$

$$\begin{cases} t_n^{(-)} \mapsto t_n^{(t)} \equiv \max\left\{ \min(t,t_n), \frac{1}{\varepsilon}\sum_{j=1}^{n-1} d_j \right\} \\ t_n^{(-)} \mapsto t_n^{(t)} \equiv \max\left\{ \min\left[\min(t,t_n), t_n + \frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n} u_j - \sum_{j=1}^{n} d_j \right], t_n \right\} \end{cases} \tag{3.2.21}$$

$$\left.\begin{aligned} \mathcal{U}_n \mapsto \mathcal{U}_n^t &\equiv \int_{t_n^{(+)}}^{t_n^{(t)}} f_n(z)\, \mathcal{P}\left(\frac{\varepsilon}{1-\varepsilon} z - \frac{1}{1-\varepsilon}\sum_{j=1}^{n-1} d_j\right) dz \\ \mathcal{D}_n \mapsto \mathcal{D}_n^t &\equiv \int_{\tau_n}^{t_n^{(t)}} f_n(z)\, \mathcal{P}\left(- z + \frac{1}{1-\varepsilon}\sum_{j=1}^{n} u_j\right) dz \end{aligned}\right\} \tag{3.2.22}$$

Then,

$$P\left\{ A_n^t \geqslant 1-\varepsilon \right\} = \begin{cases} \mathcal{U}_n^t \,/\, (U_n + D_n) & , \quad t \in [t_{n-1},\tau_n) \\ \mathcal{D}_n^t \,/\, (U_n + D_n) & , \quad t \in [\tau_n,t_n) \end{cases} \tag{3.2.23}$$

The probability distribution pf the stochastic availability for an age replacement time will be derived in § 4.2 (Eq.(4.2.9)).

The probability distributions are summarized in the following :

PROBABILITY DISTRIBUTIONS

$$P\{N_t = n\} = U_n + D_n$$

$$P\{T_{d|n} = 0\} = P\{0 \leq T_{u|n} < u_n\} = U_n/(U_n+D_n)$$

$$P\{T_{u|n} = u_n\} = P\{0 < T_{d|n} \leq d_n\} = D_n/(U_n+D_n)$$

$$P\{A_{O_n} \geq 1-\varepsilon\} = \mathcal{U}_n/(U_n+D_n)$$

$$P\{A_{M_n} \geq 1-\varepsilon\} = \mathcal{D}_n/(U_n+D_n)$$

$$P\{A_n \geq 1-\varepsilon\} = (U_n\mathcal{U}_n + D_n\mathcal{D}_n)/(U_n+D_n)^2$$

$$P\{A_c \geq 1-\varepsilon\} = \sum_{n=1}^{\infty} (U_n\mathcal{U}_n + D_n\mathcal{D}_n)/(U_n + D_n)$$

$$P\{A_n^t \geq 1-\varepsilon\} = \begin{cases} \mathcal{U}_n^t/(U_n + D_n), & t \in [t_{n-1}, \tau_n) \\ \mathcal{D}_n^t/(U_n + D_n), & t \in [\tau_n, t_n) \end{cases}$$

$$P\{A_{AR}^{t_R} \geq 1-\varepsilon\} = \sum_{n=1}^{K-1}(U_n\mathcal{U}_n+D_n\mathcal{D}_n)/(U_n+D_n) + \mathcal{U}_K + \left[1-\sum_{n=1}^{K-1}(U_n+D_n)-U_K\right]\mathcal{R}\left(\frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{K}u_j-\sum_{j=1}^{K-1}d_j\right)$$

$$U_n = \int_{t_{n-1}}^{\tau_n} f_n(z)dz \qquad D_n = \int_{\tau_n}^{t_n} f_n(z)dz$$

$$\mathcal{U}_n = \int_{t_{n-1}^{(+)}}^{\tau_n^{(-)}} f_n(z)\mathcal{R}\left(\frac{\varepsilon}{1-\varepsilon}z - \frac{1}{1-\varepsilon}\sum_{j=1}^{n-1}d_j\right)dz \; ; \; \mathcal{D}_n = \int_{\tau_n}^{t_n^{(-)}} f_n(z)\mathcal{R}\left(-z + \frac{1}{1-\varepsilon}\sum_{j=1}^{n}u_j\right)dz$$

$$\mathcal{U}_n^t = \int_{t_{n-1}^{(+)}}^{\tau_n^{(t)}} f_n(z)\mathcal{R}\left(\frac{\varepsilon}{1-\varepsilon}z - \frac{1}{1-\varepsilon}\sum_{j=1}^{n-1}d_j\right)dz \; ; \; \mathcal{D}_n^t = \int_{\tau_n}^{t_n^{(t)}} f_n(z)\mathcal{R}\left(-z + \frac{1}{1-\varepsilon}\sum_{j=1}^{n}u_j\right)dz$$

$$t_{n-1}^{(+)} = \max\left\{t_{n-1}, \frac{1}{\varepsilon}\sum_{j=1}^{n-1}d_j\right\} \; ; \; \tau_n^{(-)} = \tau_n^{(t)}\Big|_{t=\tau_n} \; ; \; t_n^{(-)} = t_n^{(t)}\Big|_{t=t_n}$$

$$\tau_n^{(t)} = \max\left\{\min(t,\tau_n), \frac{1}{\varepsilon}\sum_{j=1}^{n-1}d_j\right\}$$

$$t_n^{(t)} = \max\left\{\min\left[\min(t,t_n), t_n + \frac{\varepsilon}{1-\varepsilon}\sum_{j=1}^{n}u_j - \sum_{j=1}^{n-1}d_j\right], \tau_n\right\}$$

$\mathcal{R}(.)$ = repair time d.f.

$0 < \varepsilon < 1$

§ 3.3 ASYMPTOTIC DISTRIBUTION

We shall study the behavior of $P\{A_c \geq 1-\varepsilon\}$ as the operative-cycle time increases indefinitely (that is, no maintenance, $u_1 = \tau_1 \rightarrow \infty$). This asymptotic value will be denoted by

$$P_\infty$$

Recall from Eq.(3.2.18),

$$\begin{aligned}
P_\infty &\equiv \lim_{\tau_1 \to \infty} P\left\{A_c \geq 1-\varepsilon\right\} = \lim_{\tau_1 \to \infty} \sum_{n=1}^{\infty} (U_n \mathcal{U}_n + D_n \mathcal{D}_n)/(U_n + D_n) \\
&= \lim_{\tau_1 \to \infty} \mathcal{U}_1 && (3.3.1) \\
&= \lim_{\tau_1 \to \infty} \int_0^{\tau_1} f_1(z) \mathcal{P}\left(\frac{\varepsilon}{1-\varepsilon} z\right) dz \\
&\equiv \int_0^{\infty} \mathring{f}(t) \mathcal{P}\left(\frac{\varepsilon}{1-\varepsilon} t\right) dt && (3.3.2)
\end{aligned}$$

Since

$$\mathring{f}(t) = \lambda^\alpha \alpha t^{\alpha-1} \exp(-\lambda^\alpha t^\alpha) \qquad (3.3.3)$$

$$\mathcal{P}\left(\frac{\varepsilon}{1-\varepsilon} t\right) = 1 - \exp\left(-\frac{\mu\varepsilon t}{1-\varepsilon}\right) \qquad (3.3.4)$$

Then

$$\begin{aligned}
P_\infty &= \int_0^{\infty} \lambda^\alpha \alpha t^{\alpha-1} \exp(-\lambda^\alpha t^\alpha) \left[1 - \exp\left(-\frac{\mu\varepsilon t}{1-\varepsilon}\right)\right] dt \\
&= 1 - \int_0^{\infty} \lambda^\alpha \alpha t^{\alpha-1} \exp\left(-\frac{\mu\varepsilon t}{1-\varepsilon}\right) dt \\
&= 1 - \int_0^{\infty} \alpha z^{\alpha-1} \exp(-z^\alpha) \exp\left(-\frac{u\varepsilon}{\lambda(1-\varepsilon)} z\right) dz && (z \equiv \lambda t) \\
&= 1 - \mathcal{L}\left\{\alpha t^{\alpha-1} \exp(-t^\alpha)\right\}\Big|_{s=\frac{\mu\varepsilon}{\lambda(1-\varepsilon)}} && (3.3.5)
\end{aligned}$$

where

$$\mathcal{L}\left\{x(t)\right\} \equiv \int_0^{\infty} x(t)\exp(-st)\, dt \equiv X(s) \qquad (3.3.6)$$

is the Laplace transform, or

$$x(t) \longleftrightarrow X(s)$$

CASE 1 $\alpha = 1$

$$P_{\infty} = 1 - \mathcal{L}\left\{\exp(-t)\right\} \Big|_{s = \mu\varepsilon/\lambda/(1-\varepsilon)}$$

$$= 1 - \frac{1}{s+1} \Big|_{s = \mu\varepsilon/\lambda/(1-\varepsilon)}$$

$$= \frac{1/\lambda}{1/\lambda + [(1-\varepsilon)/\varepsilon](1/\mu)} \tag{3.3.7}$$

$$\varepsilon = \tfrac{1}{2} \quad \Longrightarrow \quad P_{\infty} = \frac{1/\lambda}{1/\lambda + 1/\mu} \tag{3.3.8}$$

$$\equiv A^{\infty} \tag{3.3.8'}$$

Eq.(3.3.7)-(3.3.8) are the same as obtained by Martz (1971) in Eq.(1.1.9) and (1.1.10).

CASE 2 $\alpha = 2$

$$P_{\infty} = 1 - \mathcal{L}\left\{2t\exp(-t^2)\right\} \Big|_{s = \mu\varepsilon/\lambda/(1-\varepsilon)} \tag{3.3.9}$$

From standard Laplace transform table,

$$\exp(-t^2) \leftrightarrow \tfrac{1}{2}\sqrt{\pi}\,\exp(\tfrac{1}{4}s^2)\,\mathrm{erfc}(\tfrac{1}{2}s) \tag{3.3.10}$$

where

$$\mathrm{erfc}(x) \equiv \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-z^2)\,dz \tag{3.3.11}$$

is the complementary error function (see e.g. Abramowitz and Stegun (1972)) Apply the differentiation theorem for Laplace transform to Eq.(3.3.10),

$$-2t\exp(-t^2) \leftrightarrow \sqrt{\pi}\,\frac{d}{ds}\left[\exp(\tfrac{1}{2}s^2)\,\mathrm{erfc}(\tfrac{1}{2}s)\right]$$

$$= \sqrt{\pi}\,\frac{d}{ds}\left\{\exp(\tfrac{1}{2}s^2)\left[1 - \frac{2}{\sqrt{\pi}}\int_0^{\frac{1}{2}s} \exp(-z^2)\,dz\right]\right\}$$

$$= \tfrac{1}{2}\sqrt{\pi}\;s\exp(\tfrac{1}{4}s^2)\,\mathrm{erfc}(\tfrac{1}{2}s)$$

$$+ \sqrt{\pi}\exp(\tfrac{1}{4}s^2)\left(-\frac{2}{\sqrt{\pi}}\exp(-\tfrac{1}{4}s^2)\right)(\tfrac{1}{2})$$

$$= -1 + \tfrac{1}{2}\sqrt{\pi}\;s\exp(\tfrac{1}{4}s^2)\,\mathrm{erfc}(\tfrac{1}{2}s) \tag{3.3.12}$$

Substituting Eq.(3.3.12) into Eq.(3.3.9),

$$P_{\infty} = \sqrt{\pi}\left(\frac{s}{2}\right)\exp\left(\left(\frac{s}{2}\right)^2\right)\operatorname{erfc}\left(\frac{s}{2}\right)\Bigg|_{s=\frac{\mu\varepsilon}{\lambda(1-\varepsilon)}}$$

$$\equiv \sqrt{\pi}\, s\exp(s^2)\operatorname{erfc}(s)\Bigg|_{s=\frac{\mu\varepsilon}{2\lambda(1-\varepsilon)}} \tag{3.3.13}$$

Some asymptotic values are tabulated in Table 3.3.1.

ε	λ	μ	s	erfc(s)	P_{∞}
0.1	0.1	1	5/9	0.43211	0.5794
0.1	0.05	1	10/9	0.11611	0.7859
0.1	0.1	2	10/9	0.11611	0.7859
0.1	0.1	1.5	5/6	0.23878	0.7063
0.15	0.1	1	15/17	0.21227	0.7231

Table 3.3.1 Some asymptotic probability values of stochastic cycle availability.

CHAPTER 4

OPTIMUM SYSTEM DESIGN

§ 4.1 PERIODIC OPERATIVE-MAINTENANCE POLICY

We have mentioned in § 2.3 that for the case of system performance and management, it is more efficient to have a periodic operative-maintenance schedule, namely, constant operative-cycle time u and constant maintenance-cycle time d (Eq.(2.1.1)). Normally, the maintenance-cycle time d is more or less fixed by the system parameters and we may free to choose an optimum operative-cycle time u^* such that certain performance index is satisfied in order to give an optimum system.

Classically, as a performance index, the minimization of a long term average operating cost or the maximization of the system availability has been proposed (Barlow and Proschan (1965)). Since we have defined availability as a random varoable, we may choose the criterion of maximizing the probability of stochastic availability with respect to the operative-cycle time to be an optimum policy. In this case we shall have a probabilistic guarantee instead of using averaged quantities. We shall see in Chapter 5 that this concept is equivalent to that of minimizing an appropriate system cost function.

If we plot $P\{A_c \geq 1-\varepsilon\}$ versus u where A_c is the stochastic cycle availability and $0 < \varepsilon < 1$, we expect the following :

(1) If there is no failure reduction, $P\{A_c \geq 1-\varepsilon\}$ increases with u because the more frequent the number of preventive maintenance, the less is the availability.

(2) If there is a failure reduction due to the maintenance operations, there may exist an u^* such that $P\{A_c \geq 1-\varepsilon\}$ is a maximum.

(3) As u increases indefinitely, there is essentially one operative cycle and no maintenance operation, therefore, (see § 3.3)

$$\lim_{u\to\infty} P\{A_c \geq 1-\varepsilon\} = P_\infty$$

The above reasoning is illustrated in Fig. 4.1.1 and the conjecture will justified by examples in § 4.4 .

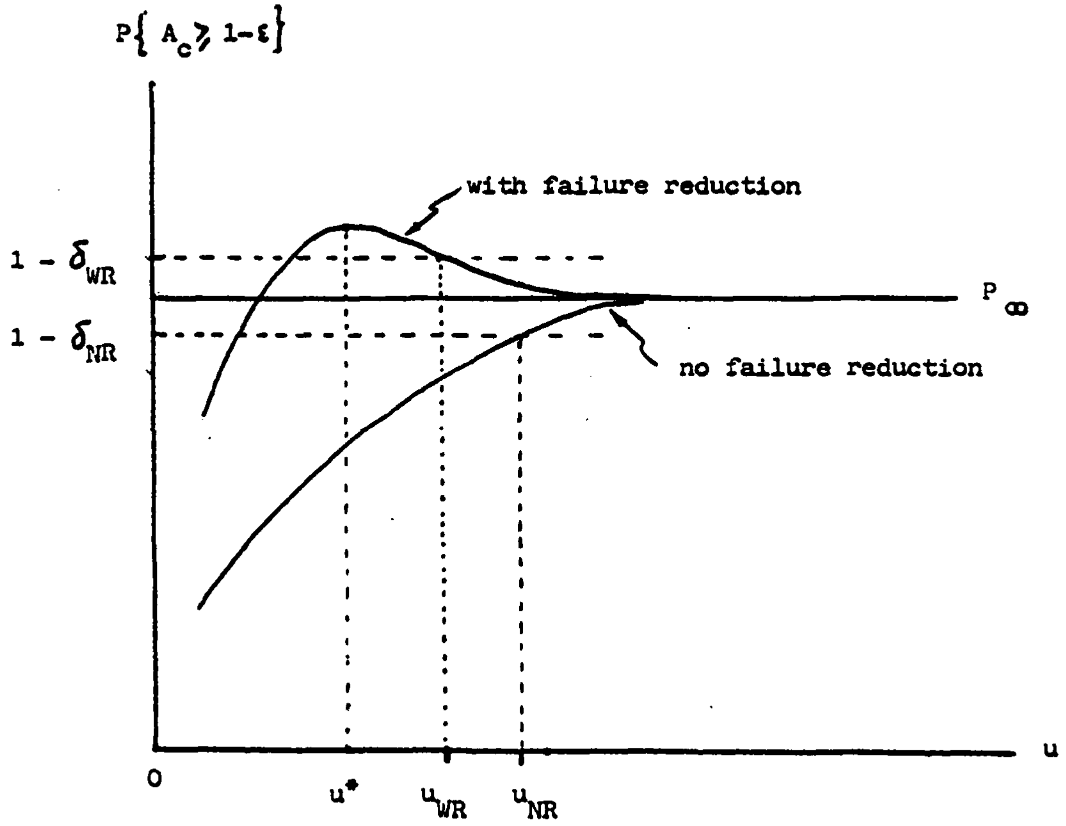


Fig. 4.1.1 Probability of stochastic cycle availability versus constant operative-cycle time for periodic O-M schedule.

From Fig. 4.1.1, we may choose an optimum operative-cycle time u in one of the following criteria :

(a) If there a failure reduction, choose

$$\left.\begin{array}{ll} & u = u^* \\ \text{where} & \max\limits_{u} \; P\left\{ A_c \geqslant 1 - \varepsilon \right\} \end{array}\right\} \qquad (4.1.1)$$

is achieved.

(b) If there is a failure reduction, choose u such that

$$\left.\begin{array}{ll} & P\left\{ A_c \geqslant 1 - \varepsilon \right\} \geqslant 1 - \delta_{WR} \quad \text{for} \quad u \geqslant u^* \\ \text{where} & \delta_{WR} = 1 - (P_{\infty} + \delta) \quad , \quad \delta > 0 \end{array}\right\} \qquad (4.1.2)$$

(c) *If there is no failure reduction, choose* u *to be the* <u>*first*</u> *time* such that the total operation cost is as small as possible, namely,

$$\left.\begin{array}{ll} & P\left\{ A_c \geqslant 1 - \varepsilon \right\} \geqslant 1 - \delta_{NR} \\ \text{where} & \delta_{NR} = 1 - (P_{\infty} - \delta) \quad , \quad \delta > 0 \end{array}\right\} \qquad (4.1.3)$$

The above criteria apply the concept of maximizing the probability of stochastic cycle availability and probabilistic inequalities in system design. Other stochastic availabilities may be used. In particular, the stochastic availability for an age replacement time will be considered in § 4.2 and the concepts will illustrated numerically in § 4.4. We shall show that these concepts are equivalent to those of minimizing the probability of an appropriate cost function and the corresponding probabilistic inequalities involving this cost function in Chapter 5.

§ 4.2 AGE REPLACEMENT POLICY

When the sytem failure rate increases with age and system failure is costly, it is usually scheduled to replace or renewal the entire system before it has aged too greatly. This is called an age replacement policy (Barlow and Proschan (1965)) which replaces the system at a time t_R after its installation or at failure, whichever occurs first. The time t_R is called the age replacement time. We shall find an age replacement time for the repairable system with an age and maintenance dependent failure rate using the concept of stochastic availability.

Suppose the age replacement time t_R is chosen to be at the end of the K^{th} operative-cycle (Fig. 4.2.1), that is, the system is replaced instead of doing preventive maintenance after the K^{th} O-cycle, where the value of K, or equivalently, t_R is to be determined. We have, from Fig. 4.2.1,

$$t_R \equiv \tau_K = t_K = \sum_{j=1}^{K} u_j + \sum_{j=1}^{K-1} d_j \tag{4.2.1}$$

$$d_K \equiv 0 \tag{4.2.2}$$

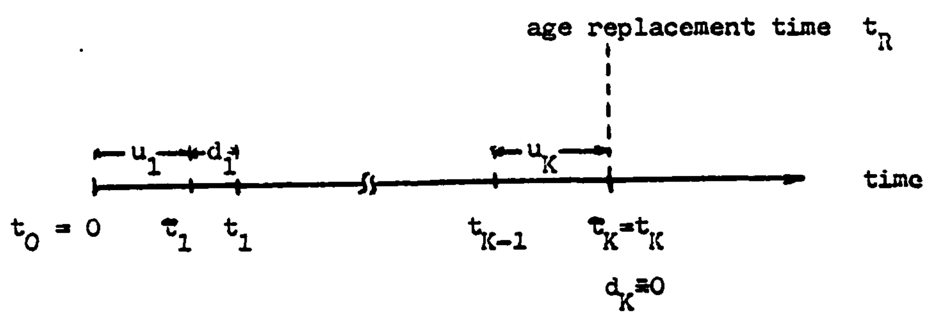


Fig. 4.2.1 An age replacement schedule.

Define <u>stochastic availability for an age replacement time</u> t_R as

$$A_{AR}^{t_R} \equiv \frac{\text{uptime before failure in } [0,t_R)}{\text{uptime before failure in } [0,t_R) \; + \; \text{total downtime}} \tag{4.2.3}$$

Note that

$$\lim_{t_R \to \infty} A_{AR}^{t_R} = A_c \tag{4.2.4}$$

Let T denotes the system lifetime, then

for $n = 1,2,3,\ldots,K$, $T \in [t_{n-1}, t_n)$

let

$$A_{AR}^{t_R}(n) \equiv \frac{\sum_{j=1}^{n-1} u_j + T_{u|n}}{\sum_{j=1}^{n-1} u_j + T_{u|n} + \sum_{j=1}^{n-1} d_j + T_{d|n} + T_F} \tag{4.2.5}$$

$$\equiv A_{N_t=n} = A_n \tag{4.2.5'}$$

where

$$d_K \equiv 0 \qquad \text{and} \qquad T_{d|K} \equiv 0 \tag{4.2.5"}$$

For $T \geq t_R$ $(= t_K = t_K)$, with $d_K \equiv 0$,

let

$$A_{AR}^{t_R}(K+1) \equiv \frac{\sum_{j=1}^{K} u_j}{\sum_{j=1}^{K} u_j + \sum_{j=1}^{K} d_j + T_F} \tag{4.2.6}$$

$$= \sum_{j=1}^{K} u_j \;/\; (\, t_R + T_F \,) \tag{4.2.6'}$$

$$\equiv A_{N_K} \qquad \text{with} \qquad T_{d|K} \equiv 0 \tag{4.2.6"}$$

Consider

$$P\left\{ A_{AR}^{t_R} \geqslant 1-\varepsilon \;\middle|\; T \geqslant t_R \right\}$$

$$= P\left\{ \sum_{j=1}^{K} u_j \,/\, (t_R + T_F) \geqslant 1-\varepsilon \right\}$$

$$= P\left\{ T_F \leqslant \frac{1}{1-\varepsilon} \sum_{j=1}^{K} u_j - t_R \right\}$$

$$= \mathscr{P}\left(\frac{1}{1-\varepsilon} \sum_{j=1}^{K} u_j - t_R \right) \tag{4.2.7}$$

$$= \mathscr{P}\left(\frac{\varepsilon}{1-\varepsilon} \sum_{j=1}^{K} u_j - \sum_{j=1}^{K-1} d_j \right) \tag{4.2.7'}$$

where

$$\mathscr{P}(t) = 1 - \exp(-\mu t) \quad , \quad t \geqslant 0$$

is the distribution function of the repair time (Eq.(2.7.2)).

Note that

$$d_K \equiv 0 \quad \Longrightarrow \quad D_K = 0 \tag{4.2.8}$$

Now we compute the probability of the stochastic availability for an age replacement time. By the total probability theorem, Eq.(4.2.5)-(4.2.8), and the probability distributions summary in § 3.2,

$$P\left\{ A_{AR}^{t_R} \geqslant 1-\varepsilon \right\}$$

$$= \sum_{n=1}^{K} P\left\{ A_{AR}^{t_R} \geqslant 1-\varepsilon \;\middle|\; N_t = n \right\} P\left\{ N_t = n \right\} + \sum_{n=K+1}^{\infty} P\left\{ A_{AR}^{t_R} \geqslant 1-\varepsilon \;\middle|\; N_t = n \right\} P\left\{ N_t = n \right\}$$

$$= \sum_{n=1}^{K} P\left\{ A_n \geqslant 1-\varepsilon \right\} P\left\{ N_t = n \right\} + \sum_{n=K+1}^{\infty} P\left\{ A_{AR}^{t_R} \geqslant 1-\varepsilon \;\middle|\; T \geqslant t_R \right\} P\left\{ N_t = n \right\}$$

$$= \sum_{n=1}^{K} \frac{U_n \mathscr{U}_n + D_n \mathscr{D}_n}{U_n + D_n} + \sum_{n=K+1}^{\infty} \mathscr{P}\left(\frac{\varepsilon}{1-\varepsilon} \sum_{j=1}^{K} u_j - \sum_{j=1}^{K-1} d_j \right) (U_n + D_n)$$

$$= \sum_{j=1}^{K-1} \frac{U_n \mathscr{U}_n + D_n \mathscr{D}_n}{U_n + D_n} + \mathscr{U}_K + \mathscr{P}\left(\frac{\varepsilon}{1-\varepsilon} \sum_{j=1}^{K} u_j - \sum_{j=1}^{K-1} d_j \right) \left[1 - \sum_{j=1}^{K} (U_n + D_n) \right]$$

$$P\left\{ A_{AR}^{t_R} \geq 1-\varepsilon \right\}$$

$$= \sum_{n=1}^{K-1} \frac{U_n \mathcal{U}_n + D_n \mathcal{D}_n}{U_n + D_n} + \mathcal{U}_K + \left[1 - \sum_{n=1}^{K-1} (U_n + D_n) - U_K \right] \mathcal{P}\left(\frac{\varepsilon}{1-\varepsilon} \sum_{j=1}^{K} u_j - \sum_{j=1}^{K-1} d_j\right) \tag{4.2.9}$$

As $t_R \to \infty$, i.e. $K \to \infty$, we have

$$\lim_{t_R \to \infty} A_{AR}^{t_R} = A_c \tag{4.2.10}$$

and

$$P\left\{ A_{AR}^{\infty} \geq 1-\varepsilon \right\} = P\left\{ A_c \geq 1-\varepsilon \right\} \tag{4.2.11}$$

For a periodic operative-maintenance schedule, we expect that the graph of $P\left\{ A_{AR}^{t_R} \geq 1-\varepsilon \right\}$ versus t_R to have the shapes shown in Fig. 4.2.2 for system with failure reduction. The curves are justified in § 4.4. These curves are similar to those in Fig. 4.1.1. Hence we propose the following age replacement policy :

(A) If the plot of $P\left\{ A_{AR}^{t_R} \geq 1-\varepsilon \right\}$ versus t_R for a given u shows a maximum, then choose

$$t_R = t_R^*$$

where

$$\max_{t_R^*} P\left\{ A_{AR}^{t_R} \geq 1-\varepsilon \right\}$$

is achieved. (4.2.12)

(B) If the plot of $P\left\{ A_{AR}^{t_R} \geq 1-\varepsilon \right\}$ versus t_R for a given u shows a maximum, then choose t_R such that

$$P\left\{ A_{AR}^{t_R} \geq 1-\varepsilon \right\} \geq 1-\delta_p \qquad \text{for} \quad t_R \geq t_R^*$$

where

$$\delta_p \equiv 1-(P_c+\delta), \quad \delta > 0$$

$$P_c \equiv P\left\{ A_c \geq 1-\varepsilon \right\} \quad \text{(for the given } u\text{)}$$

(4.2.13)

(C) if the plot $P\left\{\Lambda_{AR}^{t_R} \geq 1-\varepsilon\right\}$ versus t_R shows no peak, then choose t_R to be the first time such that

$$\left.\begin{aligned} &P\left\{\Lambda_{AR}^{t_R} \geq 1-\varepsilon\right\} \geq 1-\delta_Q \\ \text{where} \quad &\delta_Q \equiv 1-(P_c-\delta), \quad \delta > 0 \end{aligned}\right\} \qquad (4.2.14)$$

In any of the above cases, t_R is chosen to be at the end of a K^{th} 0-cycle.

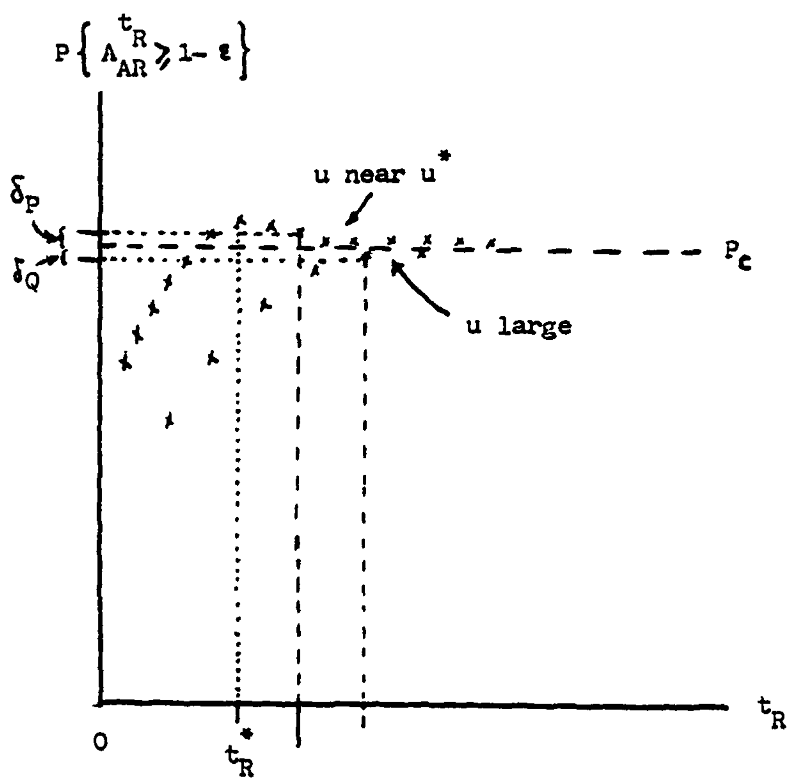


Fig. 4.2.2 Probability of age replacement stochastic availability versus age replacement time for failure reduction case.

Other types of age replacement policies can be proposed. Usually, system failure during operation is a serious problem than when it fails during maintenance. A simple age replacement policy based on this idea is stated below :

> The repairable system with maintenance schedule is replaced when the probability of system failure during operation is q, where $q \in (0,1)$, say, $q = \frac{1}{2}$.

We have from Fig. 2.6.1, the probability distributions summary in § 3.2, and the total probability theorem,

$$P\left\{\text{Failure during operation}\right\}$$

$$= \sum_{n=1}^{\infty} P\left\{\text{Failure in the } n^{th} \text{ O-M cycle} \;\middle|\; N_t=n\right\} P\left\{N_t=n\right\}$$

$$= \sum_{n=1}^{\infty} P\left\{T_{d|n} = 0\right\} P\left\{N_t=n\right\}$$

$$= \sum_{n=1}^{\infty} \frac{U_n}{U_n + D_n} (U_n + D_n)$$

$$= \sum_{n=1}^{\infty} U_n \qquad (4.2.15)$$

$$= \sum_{n=1}^{\infty} \int_{t_{n-1}}^{\tau_n} f_n(z)dz \qquad (4.2.15')$$

For an infinite operative-cycle time (no maintenance), $u \to \infty$,

$$\lim_{u \to \infty} \sum_{n=1}^{\infty} U_n = \int_0^{\infty} \overset{o}{f}(z)dz = 1 \qquad (4.2.16)$$

That is, the system will ultimately fail as expected. With the age replacement time t_R chosen to be the end of an operative-cycle, the probability curve $P\left\{\text{Failure during operation}\right\}$ versus t_R is a monotonic increasing function (Eq.(4.2.15')), and is depicted in Fig. 4.2.3.

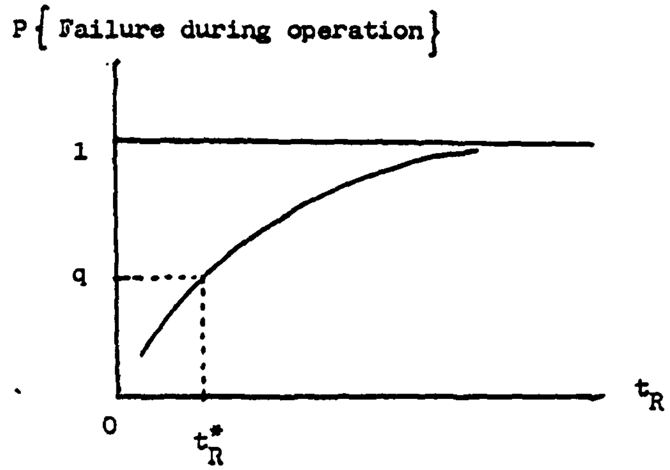


Fig. 4.2.3 Probability of failure during operation versus age replacement time.

§ 4.3 COMPUTATION

We have studied a repairable system with age and maintenance failure rates, stochastic availabilities, periodic operative-maintenance policy, and age replacement policy. We shall study various characteristics of the repairable system via examples.

In order to evaluate various probabilities of stochastic availabilities, a computer program is developed for computational purpose. The program is written in PASCAL language and is simulated by an APPLE II PLUS (1980 model) microcomputer with 48K memory.

Normally we assume that the statistics of the system failure rate function are known or can be estimated. We shall mainly concern with the periodic operative-maintenance policy, that is, fixed maintenance-cycle time (d) and constant operative-cycle time (u) . Variable operative- or maintenance- cycle times can be considered by a slight modification of the computer program.

The program has to evaluate the probability distributions of the stochastic availabilities. In particular, the stochastic cycle availability (Eq.(3.2.18)) involves an infinite summation. An upper limit of the summation is assigned to be an input to the program and the program checks the difference between two consecutive partial sums whether it is less than 10^{-7} which is the accuracy of the machine. Simpson's rule is used to evaluate all the integrals with the number of integration points as an input. The number of integration points is about 30-50 in the examples (§ 4.4). The number of summation varies from 10 to 1000 or more and it depends on the failure reduction type and failure reduction factor g . Since all quantities involved in computation are nonnegative, various checks are made to ensure this nonnegativity. Underflow or division by zero is suppressed to give correct results. Since the machine has limited accuracy and low speed, numerical difficulties occur quite frequently. If the program were written for a large computer with double precision, accuracy and speed will be improved but with a tradeoff of higher cost.

The inputs to the computer program are :

(1) ε , $0 < \varepsilon < 1$

(2) α , usually $\alpha = 2$

(3) λ_1 , λ_∞ , where $\lambda_n = \lambda_\infty - (\lambda_\infty - \lambda_1)\exp(-(n-1)/10)$, $n=1,2,\ldots$

if $\lambda_1 = \lambda_\infty$, then $\lambda_n \equiv \lambda$ = constant

(4) Reduction type :

0 = no reduction

1 = fixed reduction

2 = proportional reduction

(5) Reduction factor : $0 \leq g \leq 1$

(6) Maintenance-cycle time : d

(7) Operative-cycle time : u

(8) Maximum number of summation

(9) Integration points, e.g. 30.

The outputs from the program are :

(1) $P\{A_c \geq 1 - \varepsilon\}$

(2) $E[N_t]$

(3) t_R

(4) $P\{A_{AR}^{t_R} \geq 1 - \varepsilon\}$

(5) $P\{A_{O_n} \geq 1 - \varepsilon\}$

(6) $P\{A_{M_n} \geq 1 - \varepsilon\}$

(7) $P\{A_n \geq 1 - \varepsilon\}$

(8) $P\{N_t = n\}$

(9) Approximate mean uptime

(10) Approximate mean lifetime

A numerical illustration of the computer program is given below. A complete listing of the program is given in Appendix 2.

** STOCHASTIC AVAILABILITY **
<** FIXED REDUCTION **>
REDUCTION FACTOR = 0.50000
E = 0.10
APLHA = 2
LAMDA = 0.1000
MU = 1.0000
MAINTENANCE-CYCLE TIME = 0.01000
INTEGRATION = 36 POINTS

OPERATIVE-CYCLE TIME = 2.00000

N	P[AC>1-E]	E[N(T)]	ART	P[AAR]	P[AO]	P[AM]	P[AN]	P[N(T)=N]
1	0.005281	0.04	2.000	0.196705	0.134662	0.0018994	0.1346804	3.95958E-2
2	0.026384	0.19	4.010	0.338688	0.285096	0.0024982	0.2851142	7.45501E-2
3	0.068266	0.49	6.020	0.442228	0.416777	0.0029526	0.4167966	1.01117E-1
4	0.129460	0.96	8.030	0.514615	0.525566	0.0032888	0.5255858	1.17100E-1
5	0.204088	1.57	10.040	0.563100	0.614410	0.0035316	0.6144296	1.22117E-1
6	0.284313	2.28	12.050	0.594186	0.686677	0.0037008	0.6866968	1.17430E-1
7	0.362529	3.01	14.060	0.613240	0.745359	0.0038123	0.7453783	1.05454E-1
8	0.432853	3.73	16.070	0.624386	0.792967	0.0038789	0.7929869	8.91048E-2
9	0.491782	4.37	18.080	0.630594	0.831574	0.0039111	0.8315921	7.11892E-2
10	0.538133	4.91	20.090	0.633874	0.862872	0.0039147	0.8628907	5.39567E-2
11	0.572528	5.33	22.100	0.635509	0.888245	0.0038982	0.8882622	3.88987E-2
12	0.596692	5.66	24.110	0.636271	0.908812	0.0038655	0.9088285	2.67000E-2
13	0.612809	5.88	26.120	0.636598	0.925485	0.0038200	0.9255010	1.74857E-2
14	0.623035	6.04	28.130	0.636724	0.939002	0.0037667	0.9390177	1.09343E-2
15	0.629219	6.13	30.140	0.636764	0.949965	0.0037052	0.9499802	6.53417E-3
16	0.632786	6.19	32.150	0.636772	0.958857	0.0036409	0.9588719	3.73393E-3
17	0.634751	6.23	34.160	0.636770	0.966072	0.0035703	0.9660853	2.04153E-3
18	0.635785	6.25	36.170	0.636767	0.971926	0.0034987	0.9719387	1.06844E-3
19	0.636307	6.26	38.180	0.636764	0.976683	0.0034257	0.9766957	5.35445E-4
20	0.636558	6.26	40.190	0.636762	0.980547	0.0033518	0.9805588	2.57031E-4
21	0.636674	6.26	42.200	0.636762	0.983689	0.0032777	0.9837007	1.18218E-4
22	0.636725	6.27	44.210	0.636761	0.986246	0.0032038	0.9862578	5.21078E-5
23	0.636747	6.27	46.220	0.636761	0.988329	0.0031303	0.9883393	2.20159E-5
24	0.636755	6.27	48.230	0.636761	0.990033	0.0030587	0.9900427	8.91778E-6
25	0.636759	6.27	50.240	0.636761	0.991418	0.0029856	0.9914276	3.46367E-6
26	0.636760	6.27	52.250	0.636761	0.992559	0.0029136	0.9925676	1.29011E-6
27	0.636760	6.27	54.260	0.636761	0.993488	0.0028449	0.9934969	4.60882E-7
28	0.636761	6.27	56.270	0.636761	0.994259	0.0027774	0.9942664	1.57929E-7
29	0.636761	6.27	58.280	0.636761	0.994889	0.0027089	0.9948979	5.19149E-8

OPERATIVE-CYCLE TIME = 2.00000
P[SCA> 0.90] = 0.636761, [29]
MEAN O-M CYCLES = 6.26696
MEAN UPTIME = 1.15339E1
MEAN LIFETIME = 1.15916E1
****

Table 4.3.1 Numerical illustration of the PASCAL computer program.

§ 4.4 NUMERICAL EXAMPLES

In this section examples will be given to illustrate various concepts developed so far and to explore the characteristics of the repairable system with age and maintenance dependent failure rates. In particular, Fig. 4.1.1 and 4.2.2 are verified. The asymptotic values in Table 3.3.1 will be used in all the examples.

EXAMPLE 4.4.1

We consider the repairable system with various failure reduction criteria, namely,

(a) no reduction (NR)

(b) fixed reduction (FR)

(c) proportional reduction (PR)

for a piecewisely linear failure rate with the following parameters :

$$\alpha = 2, \quad \varepsilon = 0.1, \quad \lambda = 0.1, \quad \mu = 1, \quad d = 0.01, \quad g = 0.5$$

The failure rate functions for fixed and proportional reduction are plotted in Fig. 4.4.1. The values of $P\{A_c \geq 0.9\}$, $E[N_t]$, approximate mean uptime (Eq.(2.6.22)) and approximate mean lifetime (Eq.(2.6.23)) for (a) NR, (b) FR, and (c) PR are computed using the PASCAL computer program (Appendix 2) for various operative-cycle time u, and they are tabulated in Table 4.4.1.

$P\{A_c \geq 0.9\}$ is an increasing function of u for no reduction and it has a maximum for both types of failure reduction. For all types of failure rates $P\{A_c \geq 0.9\}$ approach the asymptotic value P_∞ (= 0.5794, see Table 3.3.1). These curves are plotted in Fig. 4.4.2 which when compared with Fig. 4.1.1, the statements made in § 4.1.1 are verified.

From appendix 1, Eq.(A1.6), the regular no failure reduction Weibull mean lifetime is

$$\frac{\sqrt{\pi}}{2\lambda} = 8.86$$

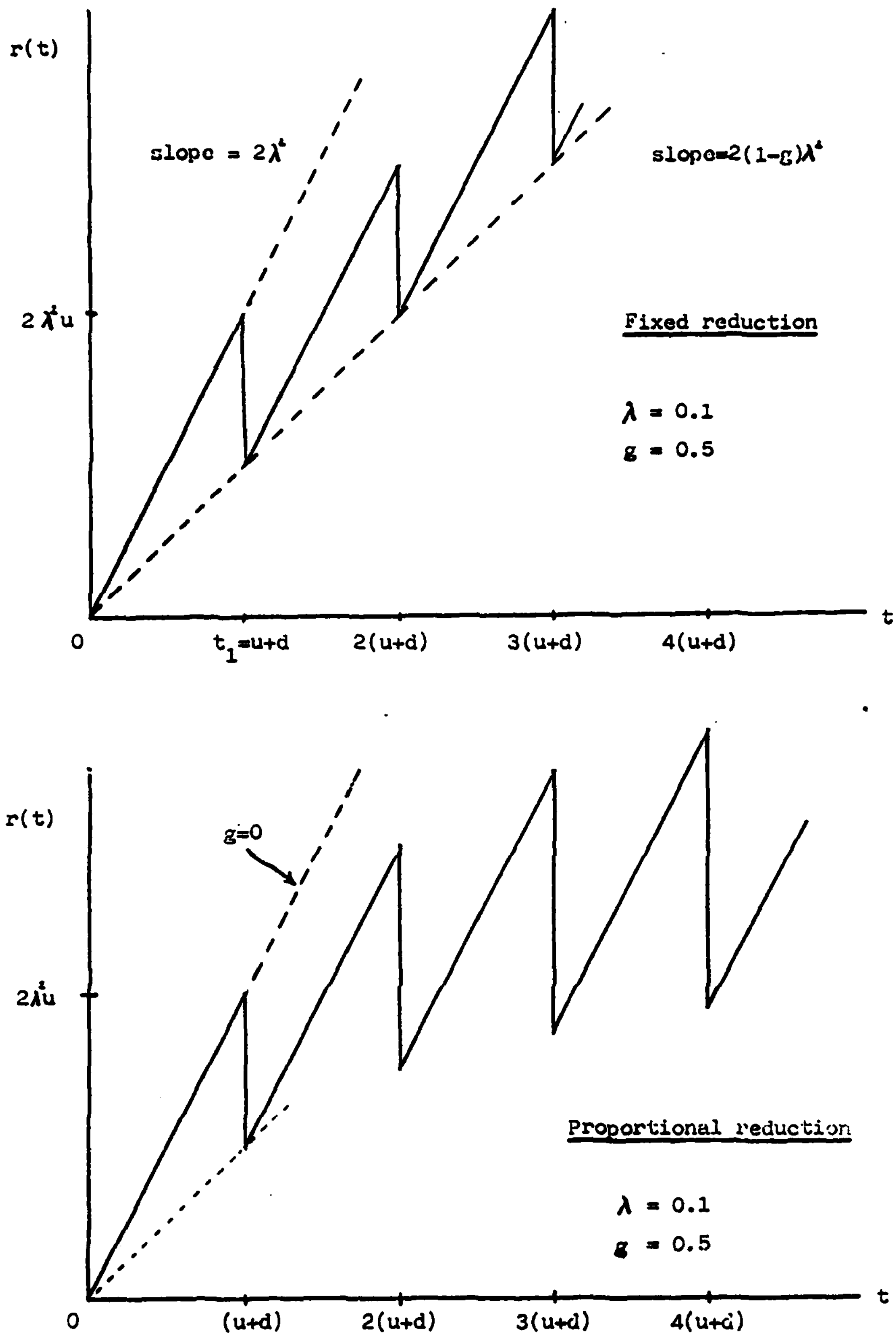


Fig. 4.4.1 Linear failure rate for fixed and proportional reduction.

$\alpha = 2, \quad \varepsilon = 0.1, \quad \lambda = 0.1, \quad d = 0.01, \quad g = 0.5, \quad \mu = 1;$ Weibull mean lifetime = 8.86.

Operative cycle time u	$P\{A_c \geq 1 - \varepsilon\}$			$E[N_t]$			Approx. mean Uptime			Approx. mean Lifetime		
	NR	FR	PR	NR	FR	PR	NR	FR	PR	NR	FR	PR
⋮												
0.5	0.4930	0.5899	0.8186	17.88	24.58	128.93	8.69	12.04	64.22	8.86	12.26	65.50
1	0.5379	0.6287	0.7763	9.27	12.42	34.48	8.77	11.92	33.98	8.86	12.04	34.32
1.5	0.5527	0.6363	0.7368	6.37	8.32	16.39	8.80	11.74	23.83	8.86	11.81	23.99
2	0.5600	0.6368	0.7066	4.91	6.27	9.98	8.82	11.53	18.95	8.86	11.59	19.04
2.5	0.5644	0.6346	0.6839	4.03	5.03	6.97	8.83	11.33	16.16	8.86	11.38	16.23
3	0.5673	0.6314	0.6664	3.44	4.21	5.30	8.83	11.13	14.40	8.86	11.17	14.45
3.5	0.5694	0.6277	0.6527	3.02	3.63	4.27	8.84	10.94	13.21	8.86	10.97	13.24
4	0.5709	0.6238	0.6418	2.71	3.19	3.59	8.84	10.76	12.35	8.86	10.78	12.38
4.5	0.5721	0.6199	0.6329	2.47	2.85	3.10	8.84	10.58	11.71	8.86	10.60	11.74
5	0.5731	0.6162	0.6255	2.27	2.58	2.74	8.84	10.41	11.22	8.86	10.43	11.24
5.5	0.5739	0.6126	0.6192	2.11	2.36	2.47	8.85	10.24	10.82	8.86	10.26	10.84
6	0.5745	0.6091	0.6139	1.97	2.18	2.25	8.85	10.09	10.50	8.86	10.11	10.52
6.5	0.5751	0.6059	0.6093	1.86	2.03	2.08	8.85	9.94	10.24	8.86	9.95	10.25
7	0.5756	0.6029	0.6053	1.76	1.90	1.93	8.85	9.81	10.01	8.86	9.82	10.03
7.5	0.5760	0.6002	0.6018	1.68	1.79	1.81	8.85	9.68	9.82	8.86	9.69	9.84
8	0.5763	0.5976	0.5988	1.61	1.70	1.71	8.85	9.56	9.66	8.86	9.57	9.67
8.5	0.5766	0.5953	0.5961	1.54	1.61	1.62	8.85	9.45	9.52	8.86	9.47	9.53
9	0.5769	0.5932	0.5937	1.48	1.54	1.54	8.85	9.36	9.40	8.86	9.37	9.41
9.5	0.5772	0.5913	0.5916	1.43	1.48	1.48	8.85	9.27	9.30	8.86	9.28	9.31
10	0.5774	0.5896	0.5898	1.39	1.42	1.42	8.85	9.19	9.21	8.86	9.20	9.22
⋮												
∞		0.5794			1			8.86			8.86	

Table 4.4.1 $P\{A_c \geq 1-\varepsilon\}$ and mean times for various operative-cycle times.

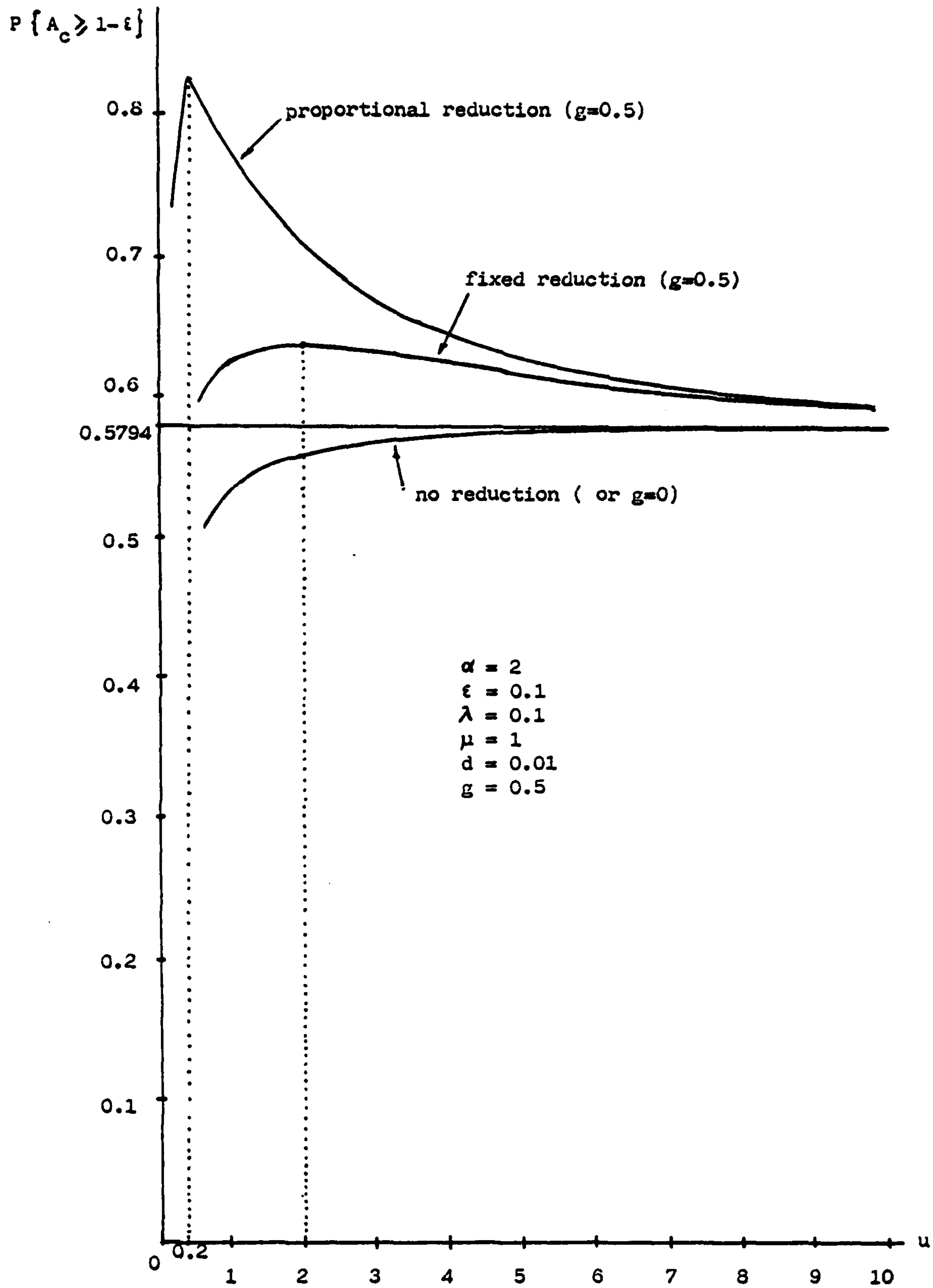


Fig. 4.4.2 $P\{A_c \geq 1-\varepsilon\}$ versus u for various reduction criteria.

This is the asymptotic value for an infinite operative-cycle time or when there is no maintenance operation (only one operative-cycle). Thus, as $u \rightarrow \infty$,

$$E[N_t] \rightarrow 1 \tag{4.4.1}$$

$$\text{Approximate mean uptime} \rightarrow \sqrt{\pi}/2/\lambda \tag{4.4.2}$$

$$\text{Approximate mean lifetime} \rightarrow \sqrt{\pi}/2/\lambda \tag{4.4.3}$$

as indicated in Table 4.4.1. The graphs of $E[N_t]$ versus u, the approximate mean uptime versus u, and the approximate mean lifetime versus u are plotted in Fig. 4.4.3, 4.4.4, and 4.4.5 respectively.

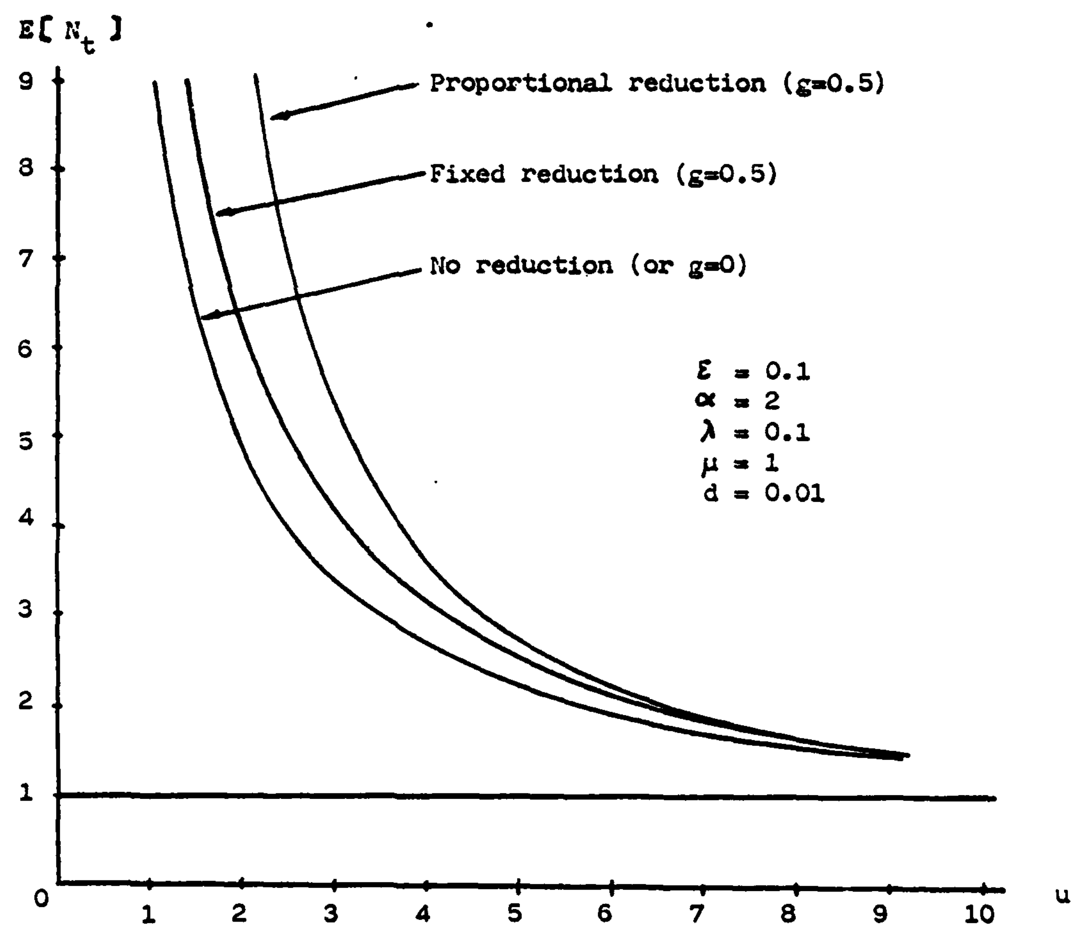


Fig. 4.4.3 Mean O-M cycles versus operative-cycle time.

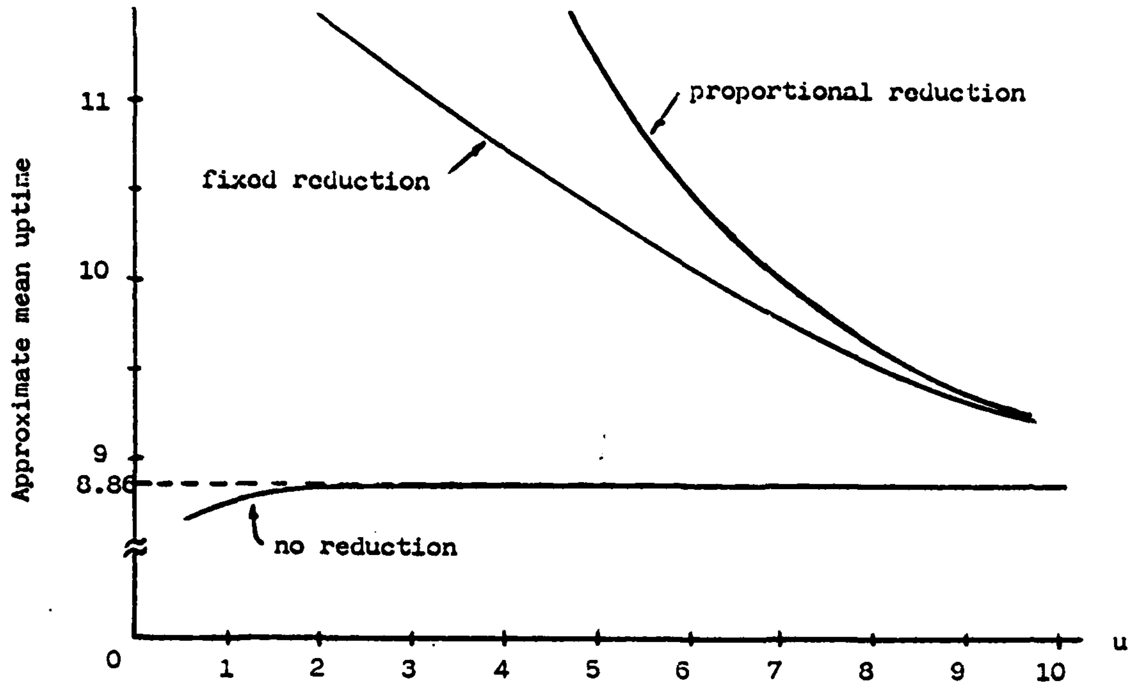


Fig. 4.4.4 Approximate mean uptime versus operative-cycle time.

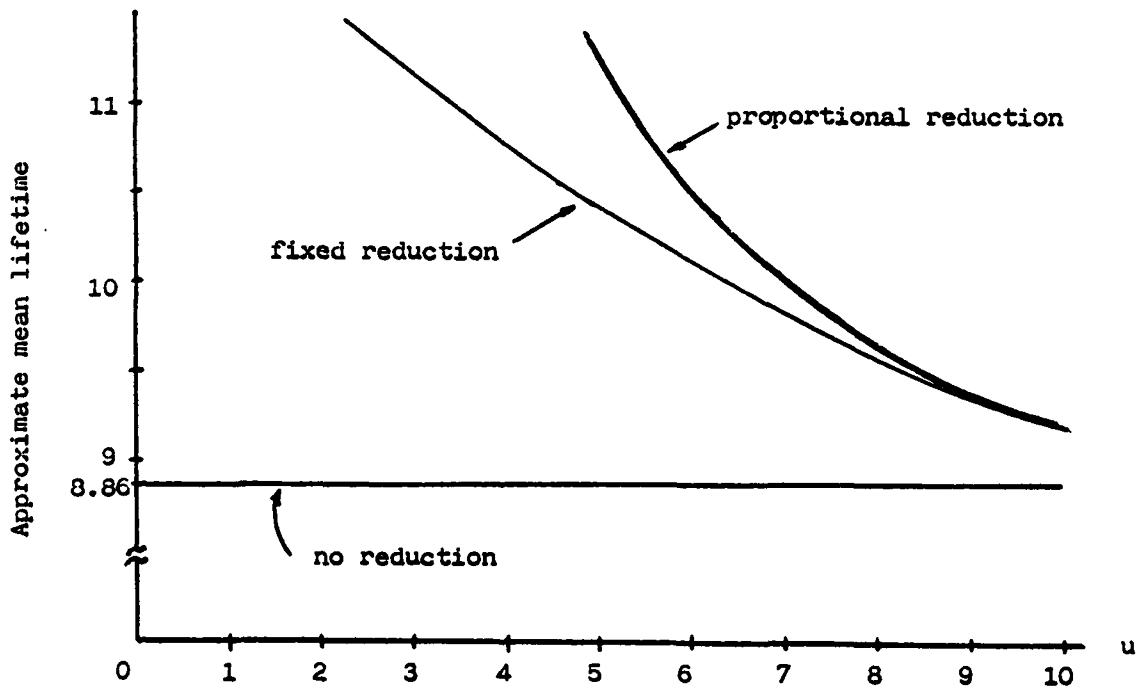


Fig. 4.4.5 Approximate mean lifetime versus operative-cycle time.

EXAMPLE 4.4.2

We shall study the characteristic of parameter variations of the repairable system with fixed reduction (proportional reduction is similar). The values of $P\{\Lambda_c \geq 1-\varepsilon\}$ versus the operative-cycle time u for $\alpha = 2$ (fixed reduction) are computed by the PASCAL computer program and they are tabulated in Table 4.4.2 for different variations of parameters :

$$\varepsilon\ ,\ \lambda\ ,\ \mu\ ,\ d\ ,\ g$$

The asymptotic values P_∞ of the stochastic cycle availabilities have been computed in Table 3.3.1. The corresponding curves are plotted.

(1) REDUCTION FACTOR VARIATION (g)

Fig. 4.4.6 depicts the variations of reduction factors $g = 0, 0.5,$ and 0.9 (columns (1), (2), and (3) in Table 4.4.2). The larger the reduction factor the more frequency of preventive maintenance is favored to improve the system. Thus the optimum operative-cycle time u^* is shorter for larger g to have more maintenance-cycles. When there is no failure reduction, that is, $g = 0$, u^* approaches infinite. Note that all curves approach asymptotically P_∞ .

(2) MAINTENANCE-CYCLE TIME VARIATION (d)

The variations of maintenance-cycle time (d) is illustrated in Fig. 4.4.7 (columns (2) and (4) in Table 4.4.2). The longer the d , the less availability of the system, hence it favors for a longer optimum operative-cycle time u^* .

u	$P\{A_c \geq 1-\varepsilon\}$ ($\alpha = 2$) Fixed reduction								
	: 0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.15
	: 0.1	0.1	0.1	0.1	0.05	.05↑.10	0.1	0.1	0.1
	u: 1	1	1	1	1	1	1.5	2	1
	d: 0.01	0.01	0.01	0.05	0.01	0.01	0.01	0.01	0.01
	g: 0	0.5	0.9	0.5	0.5	0.5	0.5	0.5	0.5
⋮									
0.5	0.4930	0.5899	0.7787	0.1139	0.7827	0.6256	0.7078	0.7797	0.7424
1	0.5379	0.6287	0.7921	0.4386	0.8195	0.7152	0.7445	0.8132	0.7675
1.5	0.5527	0.6363	0.7813	0.5189	0.8289	0.7519	0.7516	0.8197	0.7714
2	0.5600	0.6368	0.7663	0.5528	0.8318	0.7719	0.7520	0.8201	0.7702
2.5	0.5644	0.6346	0.7505	0.5701	0.8324	0.7841	0.7500	0.8185	0.7674
3	0.5673	0.6314	0.7350	0.5796	0.8318	0.7920	0.7470	0.8160	0.7639
3.5	0.5694	0.6277	0.7203	0.5849	0.8305	0.7973	0.7437	0.8131	0.7602
4	0.5709	0.6238	0.7065	0.5878	0.8290	0.8009	0.7402	0.8103	0.7565
4.5	0.5721	0.6199	0.6936	0.5892	0.8272	0.8032	0.7368	0.8074	0.7529
5	0.5731	0.6162	0.6817	0.5896	0.8254	0.8047	0.7334	0.8048	0.7495
5.5	0.5739	0.6126	0.6708	0.5894	0.8235	0.8056	0.7303	0.8023	0.7464
6	0.5745	0.6091	0.6607	0.5888	0.8216	0.8060	0.7274	0.8000	0.7434
6.5	0.5751	0.6059	0.6515	0.5880	0.8197	0.8061	0.7247	0.7979	0.7407
7	0.5756	0.6029	0.6431	0.5871	0.8178	0.8059	0.7222	0.7960	0.7383
7.5	0.5760	0.6002	0.6355	0.5861	0.8160	0.8055	0.7199	0.7943	0.7360
8	0.5763	0.5976	0.6286	0.5851	0.8142	0.8050	0.7179	0.7929	0.7340
8.5	0.5766	0.5953	0.6224	0.5842	0.8125	0.8043	0.7161	0.7916	0.7323
9	0.5769	0.5932	0.6168	0.5833	0.8108	0.8036	0.7145	0.7905	0.7307
9.5	0.5772	0.5913	0.6118	0.5824	0.8092	0.8029	0.7131	0.7896	0.7293
10	0.5774	0.5896	0.6073	0.5817	0.8077	0.8021	0.7118	0.7888	0.7281
⋮ ↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
∞	0.5794	0.5794	0.5794	0.5794	0.7859	0.7859	0.7063	0.7859	0.7231
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

Table 4.4.2 $P\{A_c \geq 1-\varepsilon\}$ versus u with variations of parameters.

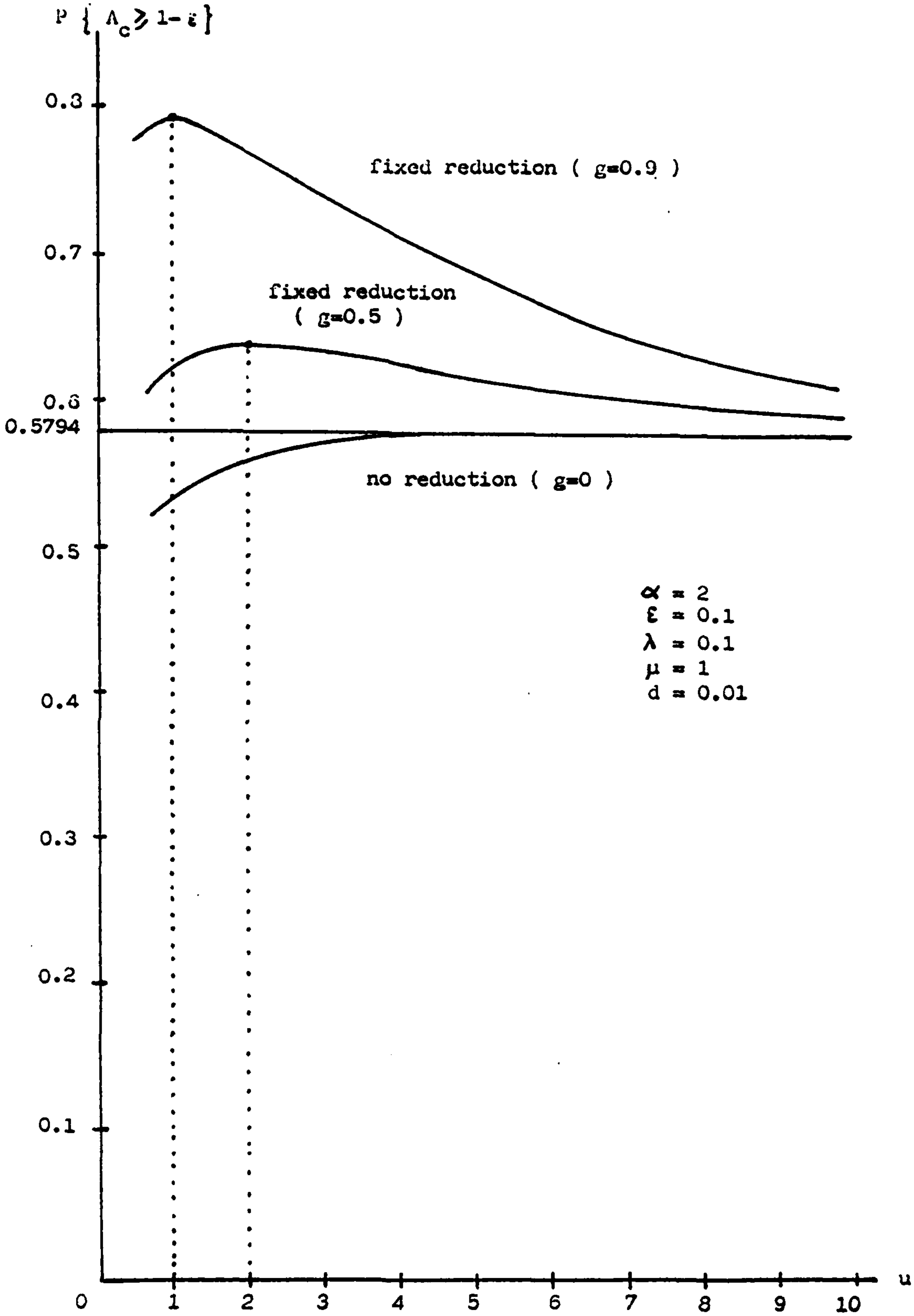


Fig. 4.4.6 $P\{A_c \geq 1-\varepsilon\}$ versus u for various reduction factors.

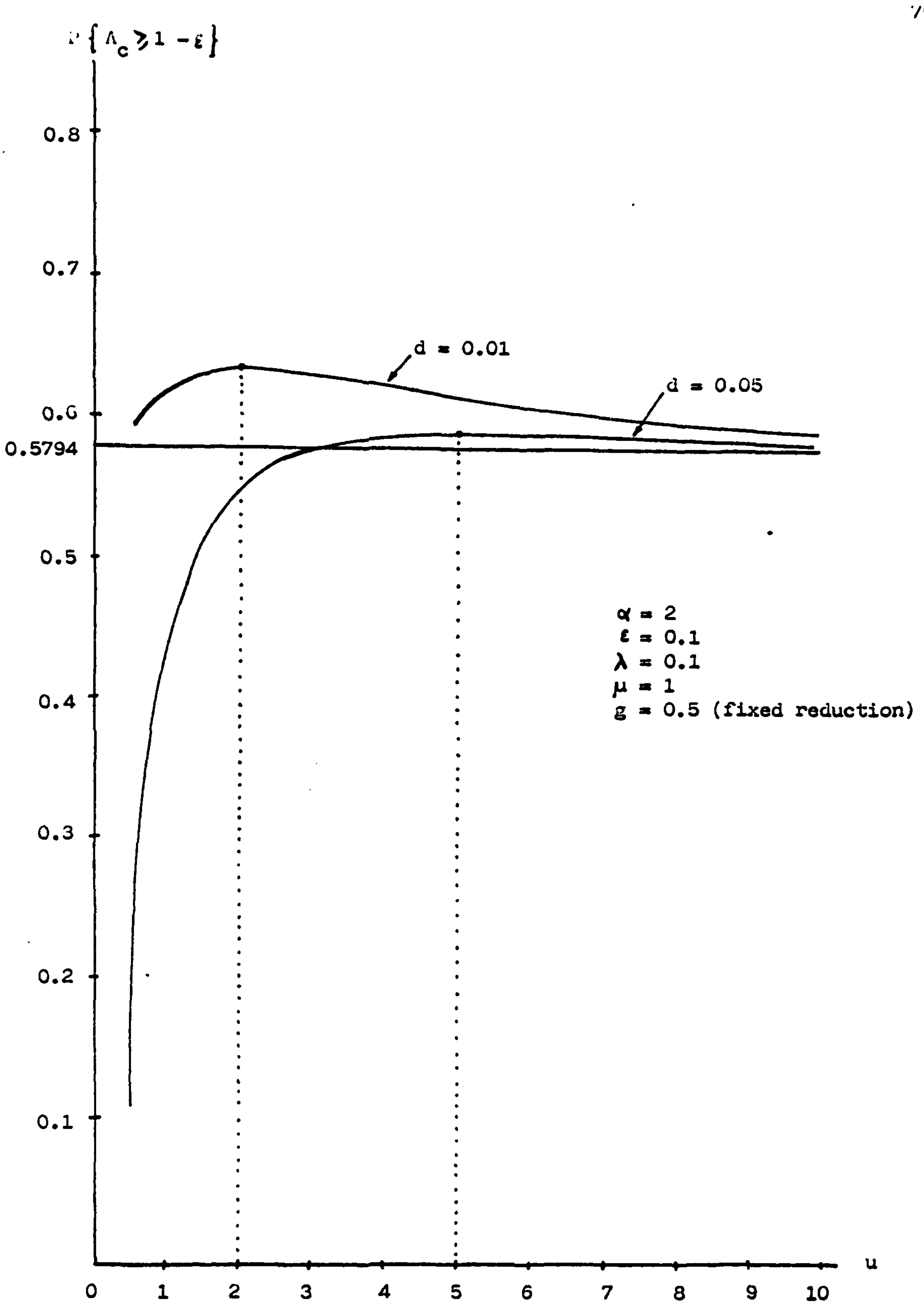


Fig. 4.4.7 $P\{A_c \geq 1-\varepsilon\}$ versus u for various maintenance-cycle times.

(3) VARIATION OF FAILURE RATE PARAMETER (λ)

The failure rate paramter λ is assumed to change after each preventive maintenance and it takes the form (other forms may be proposed) :

$$\lambda_n = \lambda_\infty - (\lambda_\infty - \lambda_1) \exp(-\frac{n-1}{10}), \quad n=1,2,\ldots \qquad (4.4.4)$$

The failure rate function r(t) with this maintenance-cycle dependent failure rate paramter λ is plotted in Fig. 4.4.8 (for fixed reduction). When λ varies from λ_1 to λ_∞, we shall write

$$\lambda_1 \uparrow \lambda_\infty$$

(as $0.05 \uparrow 0.1$ in Table 4.4.2 Column (6)). The curves of $P\{A_c \geq 1-\varepsilon\}$ versus u for $\lambda = 0.05$, 0.1 and $0.05 \uparrow 0.1$ are plotted in Fig. 4.4.9 (columns (5), (2), and (6) in Table 4.4.2). The larger the λ, the higher is the failure rate, hence it favors for more failure reduction, or a shorter optimum operative-cycle time to have more M-cycles. Thus u^* is shorter for $\lambda = 0.1$ than that for $\lambda = 0.05$. For the maintenance dependent λ, as u is small, more O-M cyles will occur, and hence the system is closer to that for $\lambda = \lambda_\infty$. As $u \to \infty$, less number of O-M cycles will take place and the system resembles the one with $\lambda = \lambda_1$. Asymptotically, the values $P_\infty = 0.5794$ and 0.7859 (from Table 3.3.1) are achieved. The optimum u^* differs greatly for the maintenance-dependent λ from the u^*'s for constant λ's. This indicates that if the failure rate parameter is not known exactly, the characteristics of the system changes greatly when the λ's are actually maintenance-dependent rather than constant.

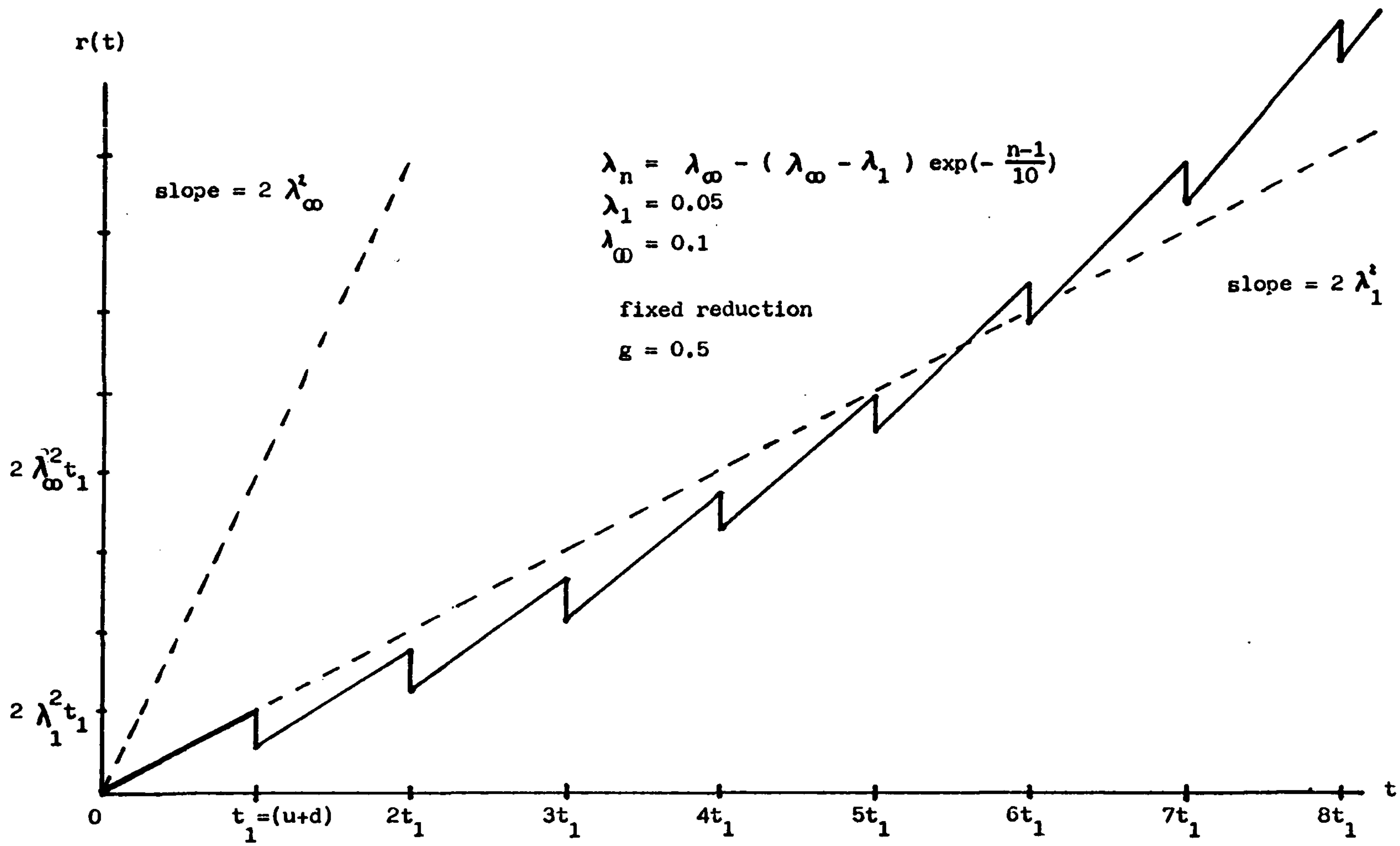


Fig. 4.4.8 Linear failure rate with maintenance-dependent failure rate parameter for fixed reduction.

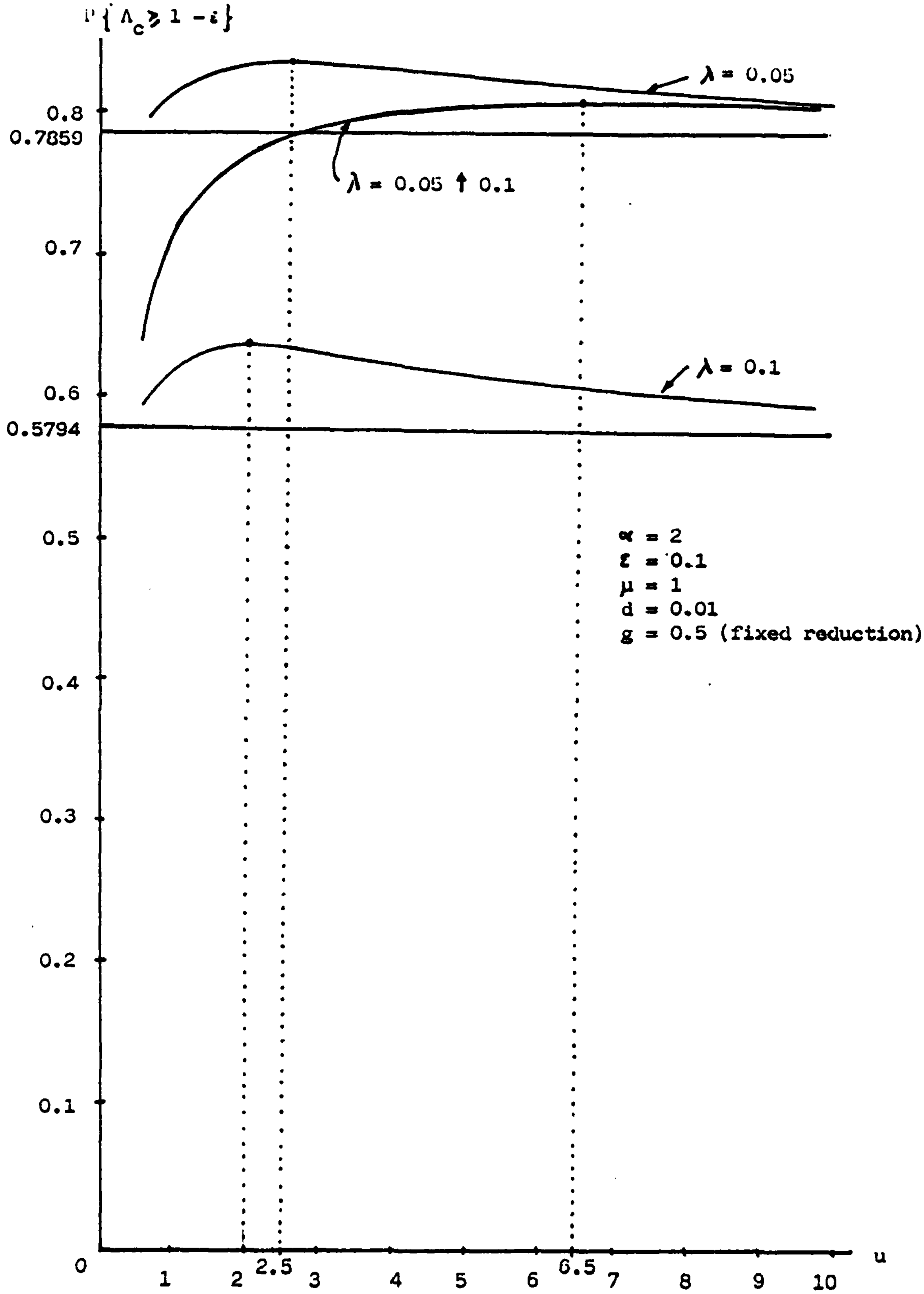


Fig. 4.4.9 $P\{\Lambda_c \geq 1-\varepsilon\}$ versus u for various failure rate parameters.

(4) VARIATION OF REPAIR RATE PARAMETER (μ)

Fig. 4.4.10 shows the variation of the repair rate parameter : μ = 1.0, 1.5, and 2.0 (columns (2), (7), and (8) in Table 4.4.2). The optimum operative-cycle time u^* does not seem to change , probably because the system failure rate is statistically independent of the repair time T_F . The smaller the μ the longer the mean repair time, hence the smaller the availability. Thus the probability curves in Fig. 4.4.10 are lower for smaller μ .

(5) VARIATION OF SYSTEM DESIGN PARAMETER (ε)

$P\{A_c \geq 0.9\}$ and $P\{A_c \geq 0.85\}$ versus u are plotted in Fig. 4.4.11 (columns (2) and (9) in Table 4.4.2). The smaller the ε the larger the A_c , hence the smaller the probability. If A_c is large, a long optimum operative-cycle time u^* has to be used. Hence u^* is larger when $\varepsilon = 0.1$ than when $\varepsilon = 0.15$.

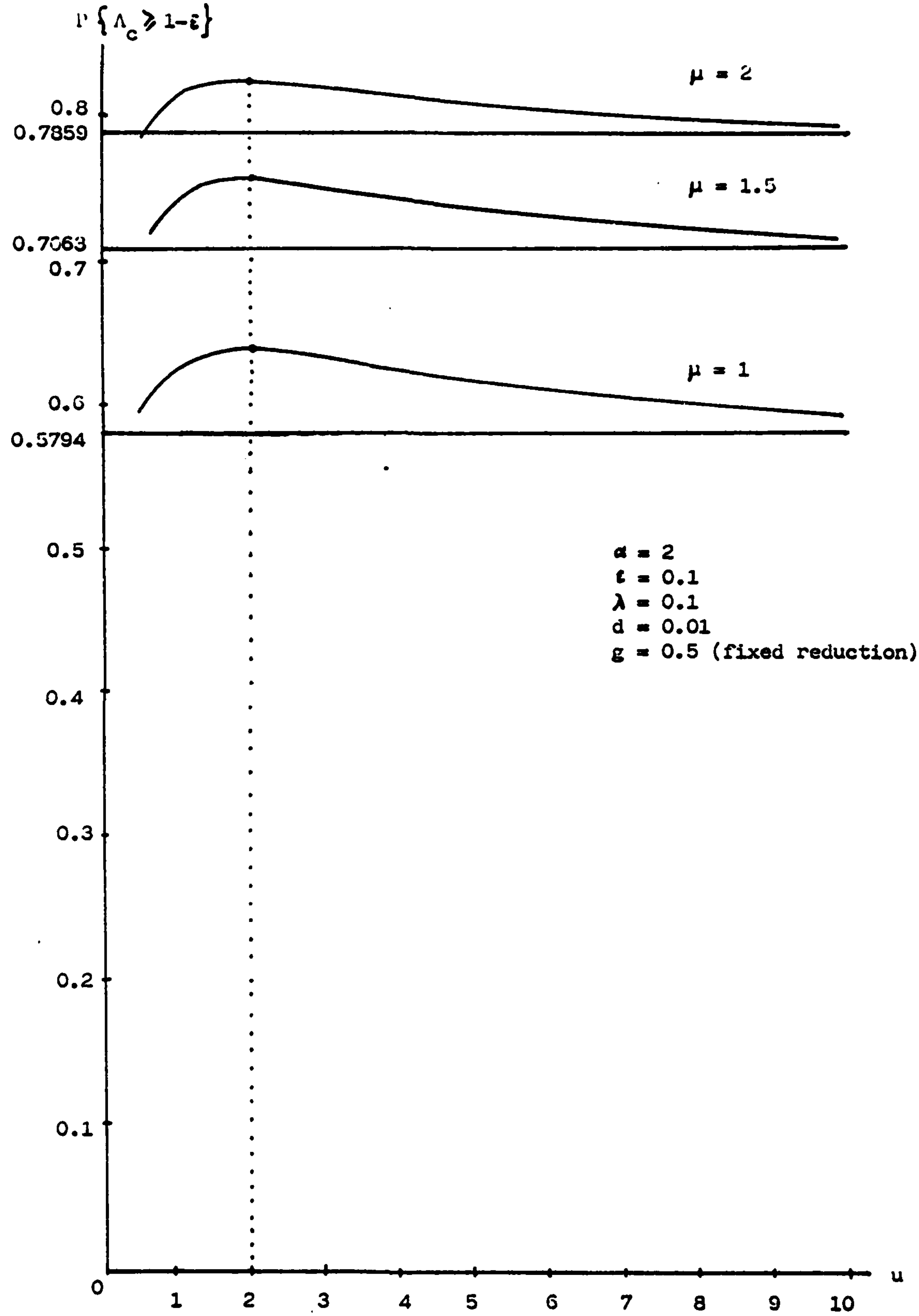


Fig. 4.4.10 $P\{A_c \geqslant 1-\varepsilon\}$ versus u for various repair rate parameters.

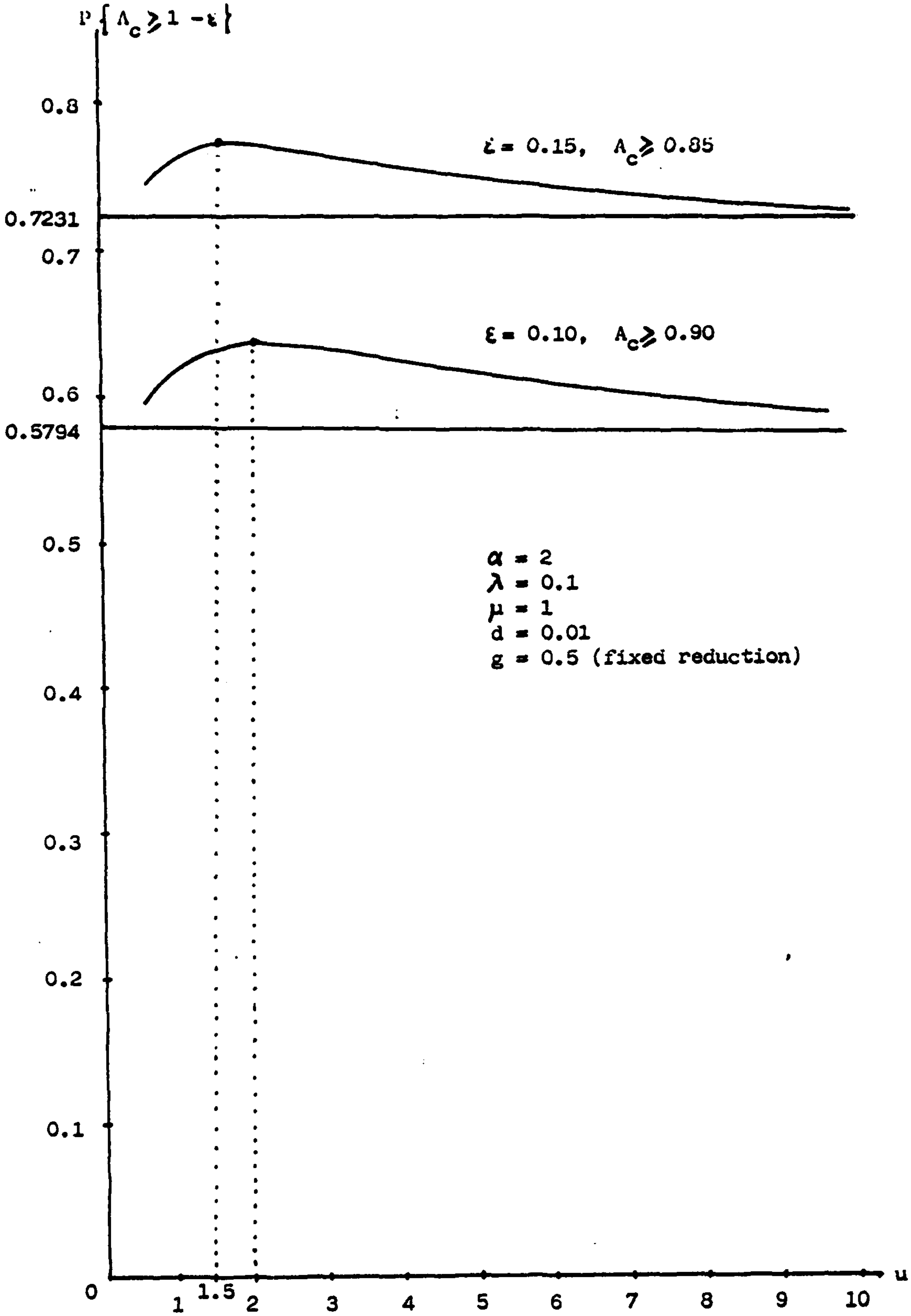


Fig. 4.4.11 $P\{A_c \geq 1-\varepsilon\}$ versus u for various design parameters ε .

EXAMPLE 4.4.3

The age replacement policy in § 4.2 will be illustrated in this example for the following case :

$\alpha = 2$, $\varepsilon = 0.1$, $\lambda = 0.1$, $\mu = 1$, $d = 0.01$, $g = .0.5$ (fixed reduction)

The values of $P\{\Lambda_c \geqslant 1-\varepsilon\}$, t_R (the age replacement time), and $P\{\Lambda_{AR}^{t_R} \geqslant 1-\varepsilon\}$ versus the operative-cycle time u are computed the PASCAL computer program.

The graph of $P\{A_{AR}^{t_R} \geqslant 1-\varepsilon\}$ versus t_R for some values of u are plotted in Fig. 4.4.12. Table 4.4.3 depicts the maximum $P\{\Lambda_{AR}^{t_R} \geqslant 1-\varepsilon\}$.

The age replacement policy is justified by comparing Fig. 4.4.2 and Fig. 4.4.12.

Fixed reduction, g=0.5 $\alpha = 2$, $\varepsilon = 0.1$, $= 0.1$, $u = 1$, $d = 0.01$			
	$P\{A_c \geqslant 1-\varepsilon\}$	t_R^*	maximum $P\{A_{AR}^{t_R} \geqslant 1-\varepsilon\}$
0.5	0.5899	25.49	0.5904
1	0.6287	28.27	0.6287
2	0.6368	32.15	0.6368
5	0.6162	35.06	0.6162
10	0.5896	40.03	0.5896

Table 4.4.3 Some optimum age replacement times.

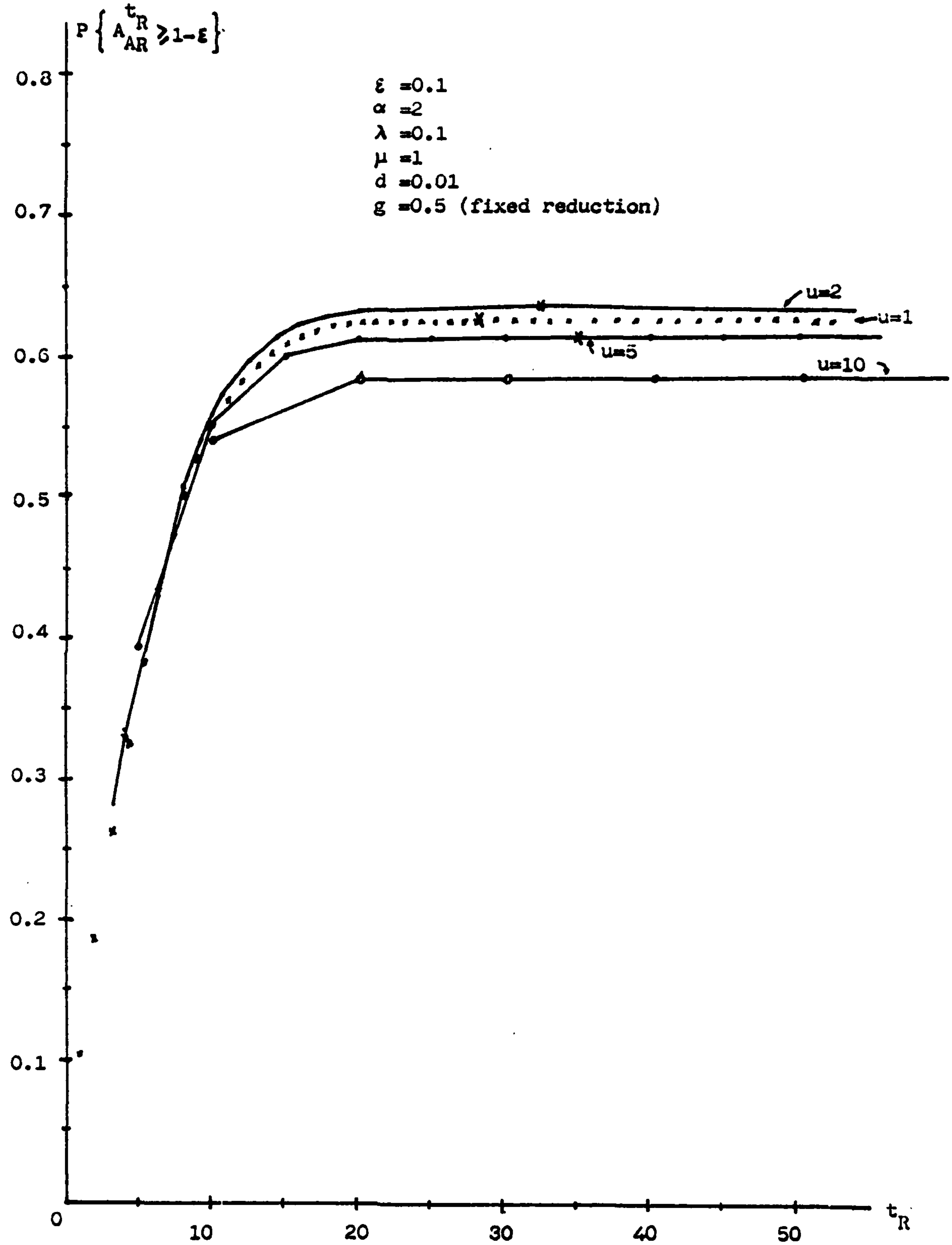


Fig. 4.4.12 $P\left\{A_{AR}^{t_R} \geq 1-\varepsilon\right\}$ versus t_R for various u .

The existence of an optimum age replacement time (Fig. 4.4.12 and Table 4.4.3) may be explained by the following argument. Suppose the system does not fail at the end of the K^{th} O-M cycle. The probability of system failure in the $(K+1)^{th}$ O-M cycle is higher than in the previous O-M cycles because of aging. Hence to replace the system at the end of the K^{th} O-cycle is more preferable than to risk over the next O-M cycle. Thus we expect an optimum t_R^* for smaller operative-cycle time u. When u is large, the system may fail during the first few O-M cycles. In this case, t_R^* should be chosen as the first time that the probability of the stochastic availability achieves certain acceptance level.(see (4.2.14)).

* * * * * * * * * *

NOTE :

The time scale in the examples is not necessary the real time unit. Interpretation is required. For example, we have

$$\lambda = 0.1 \quad \text{and} \quad u^* = 2$$

Then

$$\lambda u^* = 0.2$$

$$u^* = 0.2/\lambda$$

$$= (0.2256)\frac{\sqrt{\pi}}{2\lambda}$$

$$= (0.2256)(\text{MTBF})$$

For a real system with MTBF = 1000 hours, say, then the optimum operative-cycle time in this case is

$$u^* = 225.6 \text{ hours.}$$

Other parameters and times are interpreted similarly.

CHAPTER 5

C O S T   A N D   A V A I L A B I L I T Y

§ 5.1 GENERALIZED STOCHASTIC AVAILABILITY AND STOCHASTIC COST FUNCTION

The concept of maximizing the system availability can be interpreted as minimizing a suitable system cost function and vice versa (Barlow and Proschan (1965)).

Let C_U , C_D , and C_F be respectively the operative, maintenance, and repair costs per unit time. If T_U and T_D are some specified uptime and maintenance downtime, then the random variable

$$C_U T_U + C_D T_D + C_F T_F$$

is the total cost for a renewal cycle. The fraction

$$C \equiv \frac{C_U T_U + C_D T_D + C_F T_F}{T_U}$$

is the (random) cost per unit operative time for a renewal cycle. Let

$c_d \equiv C_D/C_U$ = relative maintenance cost with respect to operative cost

$c_f \equiv C_F/C_U$ = relative repair cost with respect to operative cost

Define the <u>stochastic cost function</u> as

$$C(c_d, c_f) \equiv \frac{T_U + c_d T_D + c_f T_F}{T_U} \qquad (5.1.1)$$

Obviously,

$$A = \frac{1}{C(1,1)}$$

is a stochastic availability.

It is therefore reasonable to define the <u>generalized stochastic availability</u> asociated with costs as

$$A(c_d,c_f) \equiv \frac{T_U}{T_U + c_d T_D + c_f T_F} \tag{5.1.2}$$

$$= 1 \,/\, C(c_d,c_f) \tag{5.1.3}$$

Note that $(0 < \varepsilon < 1 \ , \ 0 < \delta < 1)$

$$P\left\{ A(c_d,c_f) \geqslant 1 - \varepsilon \right\} = P\left\{ C(c_d,c_f) \leqslant 1 + \frac{\varepsilon}{1-\varepsilon} \right\} \tag{5.1.4}$$

or,

$$P\left\{ C(c_d,c_f) \leqslant 1 + \delta \right\} = P\left\{ A(c_d,c_f) \geqslant 1 - \frac{\delta}{1+\delta} \right\} \tag{5.1.5}$$

We may therefore define various types of generalized stochastic availabilities, or equivalently, stochastic cost functions as in §3.1 and §4.2.:

(1) <u>Generalized Stochastic Cycle Availability</u>
(<u>Stochastic Cycle Cost Function</u>)

$$A_c(c_d,c_f) \equiv \frac{1}{C_c(c_d,c_f)}$$

$$= \frac{T_{operative}}{T_{operative} + c_d T_{maintenance} + c_f T_F} \tag{5.1.6}$$

where

$T_{operative}$ = operative uptime for a renewal cycle

$T_{maintenance}$ = maintenance downtime for a renewal cycle

T_F = repair time for a renewal cycle.

(2) Generalized Stochastic Availability for the n^{th} O-M Cycle (Stochastic Cost Function for the n^{th} O-M Cycle)

$$A_{N_t=n}(c_d,c_f) \equiv \frac{1}{C_{N_t=n}(c_d,c_f)}$$

$$= \frac{\sum_{j=1}^{n-1} u_j + T_{u|n}}{\sum_{j=1}^{n-1} u_j + T_{u|n} + c_d(\sum_{j=1}^{n-1} d_j + T_{d|n}) + c_f T_F} \tag{5.1.7}$$

(3) Generalized Stochastic Availability for the n^{th} O-Cycle (Stochastic Cost Function for the n^{th} O-Cycle)

$$A_{O_n}(c_d,c_f) \equiv \frac{1}{C_{O_n}(c_d,c_f)}$$

$$= \frac{\sum_{j=1}^{n-1} u_j + T_{u|n}}{\sum_{j=1}^{n-1} u_j + T_{u|n} + c_d \sum_{j=1}^{n-1} d_j + c_f T_F} \tag{5.1.8}$$

(4) Geneneralized Stochastic Availability for the n^{th} M-Cycle (Stochastic Cost Function for the n^{th} M-Cycle)

$$A_{M_n}(c_d,c_f) \equiv \frac{1}{C_{M_n}(c_d,c_f)}$$

$$= \frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + c_d(\sum_{j=1}^{n-1} d_j + T_{d|n}) + c_f T_F} \tag{5.1.9}$$

(5) Generalized Stochastic Availability at a Finite Time in the n^{th} O-M Cycle (Stochastic Cost Function at a Finite Time in the n^{th} O-M Cycle)

$$A_n^t(c_d,c_f) \equiv \frac{1}{C_n^t(c_d,c_f)}$$

$$= \begin{cases} \dfrac{\sum_{j=1}^{n-1} u_j}{\sum_{j=1}^{n-1} u_j + \min(\hat{t}, T_{u|n}) + c_d \sum_{j=1}^{n-1} d_j + c_f T_F}, & 0 < \hat{t} = t - t_{n-1} \leq u_n \\[2ex] \dfrac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + c_d\left(\sum_{j=1}^{n-1} d_j + \min(\check{t}, T_{d|n})\right) + c_f T_F}, & 0 < \check{t} = t - \tau_n \leq d_n \end{cases} \tag{5.1.10}$$

(6) Generalized Stochastic Availability for an Age Replacement Time (Stochastic Cost Function for an Age Replacement Time)

$$A_{AR}^{t_R}(c_d,c_f) \equiv \frac{1}{C_{AR}^{t_R}(c_d,c_f)}$$

$$= \begin{cases} \dfrac{\sum_{j=1}^{n-1} u_j + T_{u|n}}{\sum_{j=1}^{n-1} u_j + T_{u|n} + c_d\left(\sum_{j=1}^{n-1} d_j + T_{d|n}\right) + c_f T_F}, & T \in [t_{n-1}, t_n) \quad n=1,2,\ldots,K \\[2ex] \dfrac{\sum_{j=1}^{K} u_j}{\sum_{j=1}^{K} u_j + c_d \sum_{j=1}^{K-1} d_j + c_f T_F}, & T \geq t_R = \tau_K = t_K \end{cases}$$

(5.1.11)

Note that when $c_d = c_f = 1$, the above definitions reduce to those defined in Chapter 3 and §4.2 .

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§ 5.2 PROBABILITY DISTRIBUTIONS OF GENERALIZED STOCHASTIC AVAILABILITY

The probability distributions of various generalized stochastic availabilities or stochastic cost functions can be derived in the same way as in Chapter 3 and § 4.2 . We shall depict Fig. 5.2.1 and Fig. 5.2.2 as parallel to Fig. 3.2.1 and Fig. 3.2.2 respectively for the domains of definitions of the corresponding variables, ans we shall summarize the expressions in the following :

$$0 < \varepsilon < 1$$

$$U_n = \int_{t_{n-1}}^{\tau_n} f_n(z)\, dz$$

$$D_n = \int_{\tau_n}^{t_n} f_n(z)\, dz$$

$$\mathcal{U}_n^t(c_d, c_f) = \int_{t_{n-1}^c}^{\tau_n^c} f_n(z)\, \mathcal{P}\left(\frac{\varepsilon/(1-\varepsilon)}{c_f} z - \frac{c_d + \varepsilon/(1-\varepsilon)}{c_f} \sum_{j=1}^{n-1} d_j\right) dz$$

$$\mathcal{D}_n^t(c_d, c_f) = \int_{\tau_n}^{t_n^c} f_n(z)\, \mathcal{P}\left(-\frac{c_d}{c_f} z + \frac{c_d + \varepsilon/(1-\varepsilon)}{c_f} \sum_{j=1}^{n-1} u_j\right) dz$$

$$t_{n-1}^c \equiv \max\left[t_{n-1} ,\ \frac{c_d + \varepsilon/(1-\varepsilon)}{\varepsilon/(1-\varepsilon)} \sum_{j=1}^{n-1} d_j \right]$$

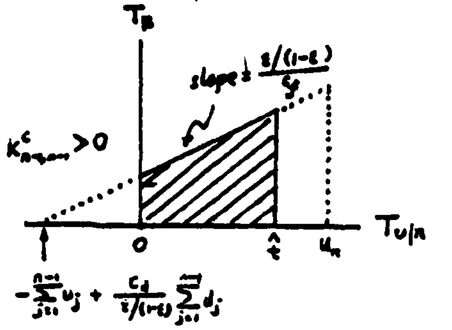
$$\tau_n^c \equiv \max\left[\min(t, \tau_n) ,\ \frac{c_d + \varepsilon/(1-\varepsilon)}{\varepsilon/(1-\varepsilon)} \sum_{j=1}^{n-1} d_j \right]$$

$$t_n^c \equiv \max\left\{ \min\left[\min(t, t_n) ,\ \frac{c_d + \varepsilon/(1-\varepsilon)}{\varepsilon/(1-\varepsilon)} \sum_{j=1}^{n} u_j \right] ,\ \tau_n \right\}$$

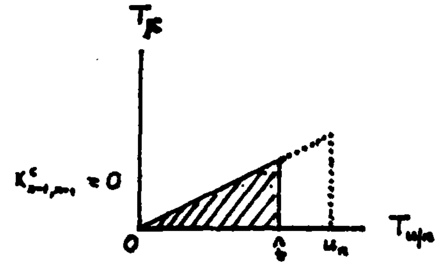
$$\mathcal{U}_n(c_d, c_f) \equiv \mathcal{U}_n^t(c_d, c_f)\Big|_{t=\tau_n}$$

$$\mathcal{D}_n(c_d, c_f) \equiv \mathcal{D}_n^t(c_d, c_f)\Big|_{t=t_n}$$

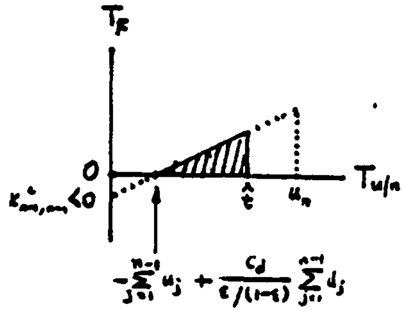
$$T_F - \frac{\varepsilon/(1-\varepsilon)}{c_f} T_{u|n} \lessgtr \underbrace{\frac{\varepsilon/(1-\varepsilon)}{c_f}\sum_{j=1}^{n-1} u_j - \frac{c_d}{c_f}\sum_{j=1}^{n-1} d_j}_{k^c_{n-1,n-1}}$$



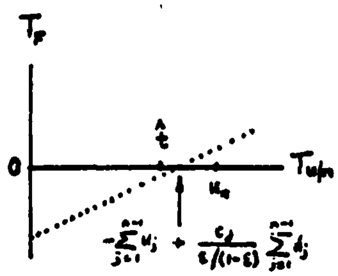
$$\frac{\sum_{j=1}^{n-1} u_j}{\sum_{j=1}^{n-1} u_j + c_d \sum_{j=1}^{n-1} d_j} > 1-\varepsilon$$



$$\frac{\sum_{j=1}^{n-1} u_j}{\sum_{j=1}^{n-1} u_j + c_d \sum_{j=1}^{n-1} d_j} = 1-\varepsilon$$



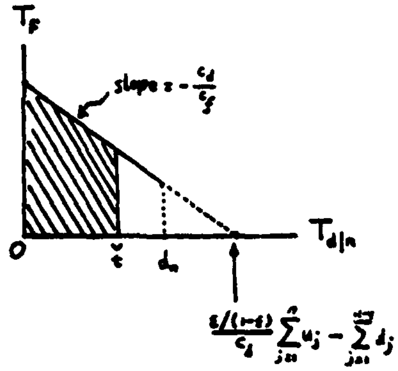
$$\frac{\sum_{j=1}^{n-1} u_j + \min(\hat{t},u_n)}{\sum_{j=1}^{n-1} u_j + \min(\hat{t},u_n) + c_d \sum_{j=1}^{n-1} d_j} > 1-\varepsilon > \frac{\sum_{j=1}^{n-1} u_j}{\sum_{j=1}^{n-1} u_j + c_d \sum_{j=1}^{n-1} d_j}$$



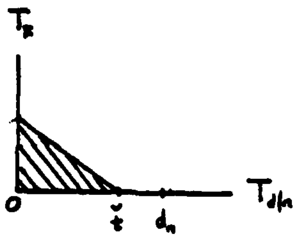
$$\frac{\sum_{j=1}^{n-1} u_j + \min(\hat{t},u_n)}{\sum_{j=1}^{n-1} u_j + \min(\hat{t},u_n) + c_d \sum_{j=1}^{n-1} d_j} \leq 1-\varepsilon$$

Fig. 5.2.1 Domain of definition of $T_{u|n}$ and T_F with costs.

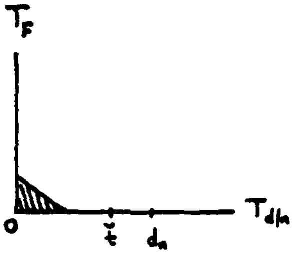
$$T_F + \frac{c_d}{c_f} T_{d|n} \leqslant \frac{\varepsilon/(1-\varepsilon)}{c_f} \sum_{j=1}^{n} u_j - \frac{c_d}{c_f} \sum_{j=1}^{n-1} d_j$$



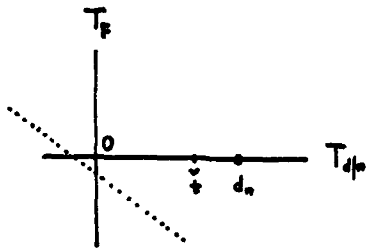
$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + c_d\left(\sum_{j=1}^{n-1} d_j + \min(\check{t}, d_n)\right)} > 1-\varepsilon$$



$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + c_d\left(\sum_{j=1}^{n-1} d_j + \min(\check{t}, d_n)\right)} = 1-\varepsilon$$



$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + c_d\left(\sum_{j=1}^{n-1} d_j + \min(\check{t}, d_n)\right)} \leqslant 1-\varepsilon < \frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + c_d \sum_{j=1}^{n-1} d_j}$$



$$\frac{\sum_{j=1}^{n} u_j}{\sum_{j=1}^{n} u_j + c_d \sum_{j=1}^{n-1} d_j} \leqslant 1-\varepsilon$$

Fig. 5.2.2 Domain of definition of $T_{d|n}$ and T_F with costs.

$$P\left\{ A_c(c_d,c_f) \geqslant 1-\varepsilon \right\} = \sum_{n=1}^{\infty} \frac{U_n \mathcal{U}_n(c_d,c_f) + D_n \mathcal{D}_n(c_d,c_f)}{U_n + D_n}$$

$$P\left\{ A_{N_t=n}(c_d,c_f) \geqslant 1-\varepsilon \right\} = \left[U_n \mathcal{U}_n(c_d,c_f) + D_n \mathcal{D}_n(c_d,c_f) \right] / (U_n + D_n)^2$$

$$P\left\{ A_{O_n}(c_d,c_f) \geqslant 1-\varepsilon \right\} = \mathcal{U}_n(c_d,c_f) \;/\; (U_n + D_n)$$

$$P\left\{ A_{M_n}(c_d,c_f) \geqslant 1-\varepsilon \right\} = \mathcal{D}_n(c_d,c_f) \;/\; (U_n + D_n)$$

$$P\left\{ A_n^t(c_d,c_f) \geqslant 1-\varepsilon \right\} = \begin{cases} \mathcal{U}_n^t(c_d,c_f) \;/\; (U_n + D_n) & , \; t \in [t_{n-1}, \tau_n) \\ \mathcal{D}_n^t(c_d,c_f) \;/\; (U_n + D_n) & , \; t \in [\tau_n, t_n) \end{cases}$$

$$P\left\{ A_{AR}^{t_R}(c_d,c_f) \geqslant 1-\varepsilon \right\} = \sum_{\substack{n=1 \\ D_K=0}}^{K} \frac{U_n \mathcal{D}_n(c_d,c_f) + D_n \mathcal{D}_n(c_d,c_f)}{U_n + D_n}$$

$$+ \left[1 - \sum_{\substack{n=1 \\ D_K=0}}^{K} (U_n + D_n) \right] \mathcal{P}\left(\frac{\varepsilon/(1-\varepsilon)}{c_f} \sum_{j=1}^{K} u_j - \frac{c_d}{c_f} \sum_{j=1}^{K-1} d_j \right)$$

CHAPTER 6

CONCLUSIONS

A general repairable system model with a periodic maintenance schedule, and age and maintenance dependent failure rate has been studied. This work examines two special types of failure reduction, various concepts of stochastic availabilities and their probability distributions, optimum system design using stochastic availability, age replacement policy, failure rate characteristics and parameter variations, computational aspects, as well as geneneralized stochastic availabilities and stochastic cost functions.

For a periodic operative-maintenance schedule with failure reduction, we have observed the existence of a unique optimum operative-cycle time u^* (Fig. 4.4.2 and 4.4.6). An analytical proof of the existence and uniqueness of u^* is difficult (Eq.(3.2.18)). However, we have studied the asymptotic behavior of the $P\{A_c \geq 1-\varepsilon\}$ versus u curve (§ 3.3 and Fig. 4.4.2). We conclude that the curve with failure reduction is above the asymptotic value P_∞ and hence above the curve with no reduction. The reason is that the lifetime probability density function with failure reduction is asymptotically greater than the p.d.f. with no reduction (Eq.(2.5.16)). Thus the $P\{A_c \geq 1-\varepsilon\}$ crosses the P_∞ asymptotically from above. Hnece the existence of an optimum (not necessary unique) u is concluded. As a further extension, the same problem with non-periodic operative-maintenance policy may be studied to prove the existence and possibly, uniqueness of a set of optimum operative-cycle times.

Some optimum system design criteria have been proposed and illustrated by examples in Chapter 4 to obtain an optimum oprative-cycle time and an age replacement time. With an appropriate interpretation of the time and parameter scales(as noted at the end of § 4.4), we can use the results and curves in § 4.4 in system design to achieve an optimum schedule and to study system characteristics against parameter variations. Since the analysis involves complicated expressions, the PASCAL computer program has found to be useful in generating results as well as insights into the problem.

Some parameter variations of the repairable system have been explored in Example 4.4.2. For a real system the statistics of the failure rate are usually unknown or can be estimated approximately only. The sensitivity of optimum design depends on parameter variations. In particular, the optimum operative-cycle time u^* changes greatly for a maintenance-dependent failure rate parameter λ than constant λ's. This affect is important in system design because this case represents a system with a rapid varying failure rate and it is therefore different from the constant failure rate parameter case. Other parameter variations can be studied in a similar way. For example, the repair rate u needs not be fixed, or the repair time may not be exponentially distributed, or the repair time may depend on the degree of system failure.

As possible extensions and generalizations, the following topics are proposed :

(1) Other types of failure reduction criteria and nonlinear failure rates (not necessarily of Weibull type) can be defined in a similar way.

(2) More general repair time distribution may be used. For repair time depends on system failure, the analysis involves the evaluation of double integrals instead of single integrals. This can be done both theoretically and numerically.

(3) Non-periodic operative-maintenance policy may be useful to obtain an optimum set of operative-cycle times by probabilistic maximization of the stochastic cycle availability when the system parameters change rapidly with age and maintenance.

(4) Age replacement policy with non-periodic operative-maintenance schedule may be treated similarly.

(5) An appropriate interpretation of system costs will apply the use of generalized stochastic availabilities or stochastic cost functions in system design.

(6) The present PASCAL computer program can be modified to satisfy all the above extensions and potential applications, because the program can be implemented, with slight changes, in all computers with the standard U.C.S.D. PASCAL language.

APPENDIX 1

W E I B U L L   D I S T R I B U T I O N

The probability density function $f(t)$ of a Weibull distribution is given by (Weibull (1951))

$$f(t) = \lambda^{\alpha} \alpha t^{\alpha-1} \exp(-\lambda^{\alpha} t^{\alpha}) \quad , \quad t \geqslant 0 \qquad \text{(A1.1)}$$

where

λ = scale parameter

α = shape parameter

The shape of $f(t)$ is sketched in Fig. A1.1.

When $\alpha = 1$, $f(t)$ is an exponential p.d.f.

When $\alpha = 2$, $f(t)$ is a Rayleigh p.d.f.

The distribution function $F(t)$ of the Weibull distribution is given by

$$F(t) = \int_0^t f(t)dt$$

$$= \begin{cases} 1 - \exp(-\lambda^{\alpha} t^{\alpha}) & , \quad t \geqslant 0 \\ 0 & , \quad t < 0 \end{cases} \qquad \text{(A1.2)}$$

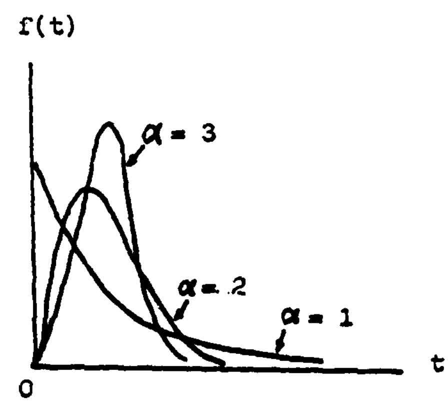


Fig. A1.1 Weibull probability density functions.

The n^{th} moment of a Weibull distribution is given by

$$\mu_n' = \int_0^\infty t^n \left[\lambda^\alpha \alpha t^{\alpha-1} \exp(-\lambda^\alpha t^\alpha) \right] dt$$

$$= \int_0^\infty \frac{1}{\lambda^n} x^{\frac{n}{\alpha}} \exp(-x)\, dx$$

$$\frac{1}{\lambda^n} \Gamma\left(\frac{n}{\alpha} + 1 \right) \qquad \text{(A1.3)}$$

where $\Gamma(.)$ is the gamma function (see e.g. Abramowitz and Stegun (1972)). In particular, the mean value is given by when $n = 1$,

$$\text{Mean} = \frac{1}{\lambda} \Gamma\left(\frac{1}{\alpha} + 1 \right) \qquad \text{(A1.4)}$$

$$\alpha = 1 \implies \text{mean} = \frac{1}{\lambda} \qquad \text{(A1.5)}$$

$$\alpha = 2 \implies \text{mean} = \frac{\sqrt{\pi}}{2\lambda} = 0.886/\lambda \qquad \text{(A1.6)}$$

The Weibull reliability function $R(t)$ is given by

$$R(t) = 1 - F(t)$$

$$= \begin{cases} \exp(-\lambda^\alpha t^\alpha) & , \quad t \geq 0 \\ 1 & , \quad t < 0 \end{cases} \qquad \text{(A1.7)}$$

The Weibull failure rate $r(t)$ is given by

$$r(t) = f(t) \,/\, R(t)$$

$$= \begin{cases} \lambda^\alpha \alpha t^{\alpha-1} & , \quad t \geq 0 \\ 0 & , \quad t < 0 \end{cases} \qquad \text{(A1.8)}$$

and the shapes of $r(t)$ are sketched in Fig. A1.2.

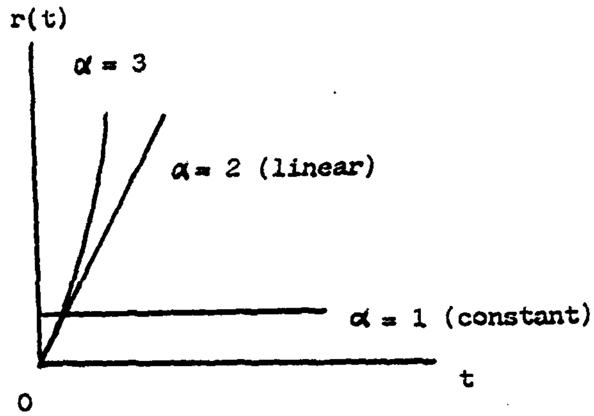


Fig. A1.2 Weibull failure rates.

APPENDIX 2

P A S C A L   C O M P U T E R   P R O G R A M

The PASCAL computer program described in § 4.3 to evaluate various probability distributions of stochastic availabilities is listed below. In the APPLE II PLUS microcomputer system the program is called the "SYSTEM.WRK.TEXT" file and is stored in a 5½" x 5½" floppy disk named the 'APPLEØ:' disk. To start the system, a disk named 'APPLE3:' is inserted into the disk drive then followed by the 'APPLEØ:' disk. One simply types "R" to run the program. The program can be implemented in other computer systems with minor modifications because the PASCAL system is the standard U.C.S.D. PASCAL language.

```
PROGRAM   CHANJACLRANG(INPUT,OUTPUT);
(*****************************************************************)
(**  PROBABILITIES OF STOCHASTIC AVAILABILITY OF A REPAIRABLE   **)
(**        SYSTEM WITH AGE AND MAINTENANCE DEPENDENT            **)
(**      FAILURE RATES AND OPTIONAL AGE REPLACEMENT POLICY      **)
(**                        APRIL 14, 1982                       **)
(*****************************************************************)
(*$S+*)

USES TRANSCEND;

CONST CONV  = 1.0E-7;
      S1    ='  N ';
      S2    ='P[AC>1-E]';
      S3    =' E[N(T)] ';
      S4    ='  ART  ';
      S5    =' P[AAR] ';
      S6    ='   P[AO] ';
      S7    ='    P[AM] ';
      S8    ='    P[AN] ';
      S9    ='   P[N(T)=N]';
      TYPE0 ='<**  NO FAILURE REDUCTION  **>';
      TYPE1 ='<**  FIXED REDUCTION  **>';
      TYPE2 ='<**  PROPORTIONAL REDUCTION  **>';
      PA    =' P[ SCA>';
      TIME=' OPERATIVE-CYCLE TIME = ';


VAR   DN,UN,UD,ID,IU,T1,T2,T3,T,TO,UPO,R,RT,RR,RF,RU,FO,
      TUP,TDOWN,TUT,TDT,F,FU,FD,DTUP,PROB,PR,DP,RD,RO,G,
      EN,MUT,MLT,E,E1,EE,DELTA,FIX,SCALE,PARAM,UPT,ART,
      AO,AM,AN,PN,AAR,MU,LAMDA,LAMAF,LINF,LZERO: REAL;
      LUB,N,ALPHA,ALFA1,POINT,TERM,KIND,K,L,
      SELECT,PRINTALL: INTEGER;
      EXIT: BOOLEAN;
      CHOICE,DATE: STRING;
      FILER: INTERACTIVE;


PROCEDURE OUTPUTCONTROL;
BEGIN
  WRITELN('OUTPUT MEDIUM :');
  WRITELN('  1 = CONSOLE');
  WRITELN('  2 = PRINTER');
  WRITE('WHICH NUMBER? '); READLN(SELECT);
  CASE SELECT OF
    1: CHOICE:='CONSOLE:';
    2: CHOICE:='PRINTER:';
    END;
  IF (SELECT=2) THEN
    BEGIN
      WRITE('  PRINT ALL ? (YES=1, NO=0) ');READLN(PRINTALL)
    END;
END; (* OUTPUT CONTROL *)


FUNCTION POWER(X: REAL; Y: INTEGER): REAL;
VAR Z: REAL;
BEGIN
  Z:=1.0;
```

```
  WHILE Y>0 DO
    BEGIN
      WHILE NOT ODD(Y) DO
        BEGIN
          Y:=Y DIV 2;
          X:=X*X
        END;
      Y:=Y-1;
      Z:=X*Z
    END;
    POWER:=Z
END; (* INTEGRAL POWER *)


FUNCTION MAX(A: REAL; B: REAL): REAL;
BEGIN
  MAX:=0.5*(A+B+ABS(A+B))
END; (* MAXIMUM *)


FUNCTION MIN(A: REAL; B: REAL): REAL;
BEGIN
  MIN:=0.5*(A+B-ABS(A-B))
END; (* MINIMUM *)


FUNCTION EXPF(T: REAL): REAL;
VAR X: REAL;
BEGIN
  IF(T<-60.0)THEN X:=0.0
             ELSE X:=EXP(T);
  EXPF:=X
END; (* EXPONENTIAL FUNCTION WITH UNDERFLOW SUPPRESSION *)


FUNCTION REPAIR(T: REAL): REAL;
BEGIN
  REPAIR:=1.0-EXPF(-MU*T)
END; (* EXPONENTIAL REPAIR TIME CDF *)


PROCEDURE DISTRIBUTION(T,TO,RR,DEL: REAL;VAR R,RF,F: REAL);
BEGIN
  R :=PARAM*POWER(T,ALFA1)-DELTA;
  RF:=EXPF(-LAMAF*(POWER(T,ALPHA)-POWER(TO,ALPHA))+DELTA*(T-TO));
  F :=R*RF*RR
END; (* AGE & MAINTENANCE DEPENDENT WEIBULL FAILURE RATE *)


PROCEDURE SIMPSON(T1,I1,T2,I3,RR,DELTA,EE,TTU,TTD,HU,HD: REAL;
                  NPU,NPD: INTEGER; VAR IU,ID: REAL);
VAR SU0,SU1,SU2,SU4,SD0,SD1,SD2,SD4,PDF,SF,
    X,Y,U1,U2,D1,D2: REAL;
    J,NU1,ND1: INTEGER;
BEGIN
  IU:=0.0;
  IF HU>0.0 THEN
    BEGIN
      U1 :=MAX(0.0,EE*I1-TTD);
      DISTRIBUTION(I1,T1,RR,DELTA,R,SF,PDF);
```

```
        SU0:=PDF*REPAIR(U1);
        U2 :=EE*T2-TTD;
        DISTRIBUTION(T2,T1,RR,DELTA,R,SF,PDF);
        SU1:=PDF*REPAIR(U2);
        SU2:=0.0;
        SU4:=0.0;
        NU1:=NPU-1;
        FOR J:=1 TO NU1 DO
          IF ODD(J) THEN
            BEGIN
              X   :=I1+J*HU;
              Y   :=EE*X-TTD;
              DISTRIBUTION(X,T1,RR,DELTA,R,SF,PDF);
              SU4:=SU4+PDF*REPAIR(Y)
            END
          ELSE
            BEGIN
              X   :=I1+J*HU;
              Y   :=EE*X-TTD;
              DISTRIBUTION(X,T1,RR,DELTA,R,SF,PDF);
              SU2:=SU2+PDF*REPAIR(Y)
            END;
        IU:=HU*(SU1+SU0+2.0*SU2+4.0*SU4)/3.0;
      END;
    ID:=0.0;
    IF HD>0.0 THEN
      BEGIN
        D1 :=-T2+TTU;
        DISTRIBUTION(T2,T1,RR,DELTA,R,SF,PDF);
        SD0:=PDF*REPAIR(D1);
        D2 :=-I3+TTU;
        DISTRIBUTION(I3,T1,RR,DELTA,R,SF,PDF);
        SD1:=PDF*REPAIR(D2);
        SD2:=0.0;
        SD4:=0.0;
        ND1:=NPD-1;
        FOR J:=1 TO ND1 DO
          IF ODD(J) THEN
            BEGIN
              X   :=T2+J*HD;
              Y   :=-X+TTU;
              DISTRIBUTION(X,T1,RR,DELTA,R,SF,PDF);
              SD4:=SD4+PDF*REPAIR(Y)
            END
          ELSE
            BEGIN
              X   :=T2+J*HD;
              Y   :=-X+TTU;
              DISTRIBUTION(X,T1,RR,DELTA,R,SF,PDF);
              SD2:=SD2+PDF*REPAIR(Y)
            END;
      ID:=HD*(SD1+SD0+2.0*SD2+4.0*SD4)/3.0;
    END;
  END; (* SIMPSON'S RULE *)


  PROCEDURE INTEGRATION(T1,T2,T3,RR,DELTA: REAL;
                        VAR UPT,IU,ID: REAL);
  VAR HD,HU,I1,I3,TTU,TTD,UTT: REAL;
      NP,NPU,NPD: INTEGER;
```

```
BEGIN
  NPU:=2*ROUND(POINT/2);
  NPD:=2*ROUND(POINT/4);
  I1 :=MIN(T2,T1+MAX(0.0,-TUT+TDT/EE));
  UTT:=TUT+T2-T1;
  I3 :=MAX(0.0,MIN(T3,T2+EE*UTT-TDT));
  TTU:=UTT/E1;
  TTD:=TDT/E1;
  HU :=MAX(0.0,(T2-I1)/NPU);
  HD :=MAX(0.0,(I3-T2)/NPD);
  SIMPSON(T1,I1,T2,I3,RR,DELTA,EE,TTU,TTD,HU,HD,NPU,NPD,IU,ID);
  UPT:=UTT;
  IU :=MAX(0.0,IU);
  ID :=MAX(0.0,ID)
END; (* INTEGRATION *)


PROCEDURE WEIBULL(T1: REAL; VAR RO: REAL);
BEGIN
  LAMDA:=LINF-(LINF-LZERO)*EXP(-(N-1)/SCALE);
  ALFA1:=ALPHA-1;
  LAMAF:=POWER(LAMDA,ALPHA);
  PARAM:=ALPHA*LAMAF;
  RO    :=PARAM*POWER(T1,ALFA1)
END; (* WEIBULL PARAMETER *)


PROCEDURE PROBABILITY;
BEGIN
  PR    :=PROB;
  DISTRIBUTION(T2,T1,RR,DELTA,R,RU,FU);
  DISTRIBUTION(T3,T1,RR,DELTA,RT,RD,FD);
  UN    :=MAX(0.0,RR*(1.0-RU));
  DN    :=MAX(0.0,RR*(RU-RD));
  UD    :=DN+UN;
  PN    :=PN-UD;
  FO    :=FO+UN;
  INTEGRATION(T1,T2,T3,RR,DELTA,UPT,IU,ID);
  IF (UD>0.0) THEN
    BEGIN
      AO    :=IU/UD;
      AM    :=ID/UD;
      AN    :=UN*AO+DN*AM;
      PROB  :=PROB+AN;
      AN    :=AN/UN
    END;
  ART   :=UPT+TDT;
  AAR   :=PR+IU+PN*REPAIR(UPT/E1-ART);
  EN    :=EN+N*UD;
  IF (PRINTALL=1) AND (SELECT=2) THEN
    WRITELN(FILER,N:3,PROB:10:6,EN:7:2,ART:8:3,
            AAR:9:6,AO:10:6,AM:10:7,AN:10:7,UD:12);
  WRITELN(N:3,PROB:10:6,EN:7:2,ART:8:3,AAR:9:6,
          AO:10:6,AM:10:7,AN:10:7,UD:12);
  TERM :=N;
  TUT   :=TUT+T2-T1;
  TDT   :=TDT+T3-T2;
  RR    :=RR*RD;
  T1    :=T3;
  T2    :=T1+TUP;
```

```
  T3    :=T2+TDOWN;
  N     :=N+1;
  WEIBULL(T1,RO);
  IF (KIND=1) THEN
    DELTA :=MAX(0.0,RO-RT+FIX);
  IF (KIND=2) THEN
    DELTA :=MAX(0.0,RO-(1.0-G)*RT);
  DP    :=ABS(PROB-PR);
  IF (DP<CONV) AND (PROB<1.0) AND (N>1) THEN EXIT:=TRUE
END; (* TOTAL PROBABILITY *)


PROCEDURE INPUT;
BEGIN
  WRITELN;
  WRITE('****  DATE :  ');READLN(DATE);
  WRITE('             E = ');READLN(E);
  WRITE('         ALPHA = ');READLN(ALPHA);
  WRITE('LAMDA INITIAL = ');READLN(LZERO);
  WRITE('LAMDA  FINAL  = ');READLN(LINF);
  SCALE :=10.0;
  WRITE('            MU = ');READLN(MU);
  WRITE('MAINTENANCE-CYCLE TIME = ');READLN(TDOWN);
  WRITELN('CRITERION OF FAILURE REDUCTION : ');
  WRITELN('  TYPE 0 = NO FAILURE REDUCTION ');
  WRITELN('  TYPE 1 = FIXED REDUCTION ');
  WRITELN('  TYPE 2 = PROPORTIONAL REDUCTION ');
  WRITE('              TYPE  = ');READLN(KIND);
  IF (KIND=0) THEN  G:=0.0
    ELSE
      BEGIN
       WRITE('  REDUCTION FACTOR = ');READLN(G);
      END;
  WRITE('  INITIAL O-CYCLE TIME = ');READLN(UPO);
  WRITE('O-CYCLE TIME INCREMENT = ');READLN(DTUP);
  WRITE('NUMBER OF O-CYCLE TIME = ');READLN(L);
  WRITE('   SUMMATION TERMS = ');READLN(LUB);
  WRITE('INTEGRATION POINTS = ');READLN(POINT)
END; (* INPUT DATA *)


PROCEDURE DATA;
BEGIN
  INPUT;
  LAMDA:=LZERO;
  E1    :=1.0-E;
  EE    :=E/E1;
  REWRITE(FILER,CHOICE);
  WRITELN(FILER);
  WRITELN(FILER,'DATE : ',DATE);WRITELN(FILER);
  WRITELN(FILER,'**  STOCHASTIC AVAILABILITY  **');
  IF (KIND=0) THEN WRITELN(FILER,TYPE0)
    ELSE
      BEGIN
        IF (KIND=1) THEN WRITELN(FILER,TYPE1);
        IF (KIND=2) THEN WRITELN(FILER,TYPE2);
        WRITELN(FILER,'      REDUCTION FACTOR = ',G:7:5)
      END;
  WRITELN(FILER,'             E = ',E:5:2);
  WRITELN(FILER,'         APLHA = ',ALPHA:2);
```

```
  IF (LZERO=LINF) THEN
      WRITELN(FILER,'           LAMDA = ',LAMDA:7:4)
    ELSE
      BEGIN
        WRITELN(FILER,'LAMDA INITIAL = ',LZERO:7:4);
        WRITELN(FILER,'LAMDA  FINAL  = ',LINF:7:4)
      END;
  WRITELN(FILER,'            MU = ',MU:7:4);
  WRITELN(FILER,'MAINTENANCE-CYCLE TIME = ',TDOWN:7:5);
  WRITELN(FILER,'           INTEGRATION = ',POINT,' POINTS')
END;(* PRINT DATA *)


PROCEDURE HEADING;
BEGIN
  IF (PRINTALL=1) AND (SELECT=2) THEN
    BEGIN
      WRITELN(FILER);
      WRITELN(FILER,TIME,TUP:8:5);
      WRITELN(FILER,S1,S2,S3,S4,S5,S6,S7,S8,S9)
    END;
  WRITELN( );
  WRITELN('.....',TIME,TUP:8:5);
  WRITELN(S1,S2,S3,S4,S5,S6,S7,S8,S9);
  WRITELN( )
END; (* PRINT HEADING *)


PROCEDURE INITIAL;
BEGIN
  EXIT :=FALSE;
  ALFA1:=ALPHA-1;
  LAMAF:=POWER(LZERO,ALPHA);
  PARAM:=ALPHA*LAMAF;
  FIX  :=G*PARAM*(TUP+TDOWN);
  DELTA:=0.0;
  N    :=1;
  PROB :=0.0;
  PN   :=1.0;
  RR   :=1.0;
  FO   :=0.0;
  TDT  :=0.0;
  TUT  :=0.0;
  EN   :=0.0;
  T1   :=0.0;
  T2   :=TUP;
  T3   :=T2+TDOWN
END; (* INITIAL CONDITIONS *)


BEGIN (** MAIN PROGRAM **)
  OUTPUTCONTROL;
  DATA;
  TUP:=UP0;
  FOR K:=1 TO L DO
    BEGIN
      INITIAL;
      HEADING;
      REPEAT  PROBABILITY  UNTIL (N>LUB) OR EXIT;
      IF N>LUB THEN
```

```
        BEGIN
          WRITELN(FILER);
          WRITELN(FILER,'** SCA NOT CONVERGE IN ',LUB,' TERMS **');
          WRITELN(FILER,PA,E1:4:2,' ] = ',PROB:9:6,', ',TIME,TUP)
        END
      ELSE
        BEGIN
          MUT:=TUP*(EN-0.5);
          MLT:=(TUP+TDOWN)*(EN-0.5);
          WRITELN(FILER);WRITELN(FILER,TIME,TUP:8:5);
          WRITELN(FILER,PA,E1:4:2,' ] = ',PROB:9:6,', [',TERM,']');
          WRITELN(FILER,'MEAN O-M CYCLES = ',EN);
          WRITELN(FILER,'     MEAN UPTIME = ',MUT);
          WRITELN(FILER,'  MEAN LIFETIME = ',MLT)
        END;
      TUP:=TUP+DTUP
    END;
  WRITELN();WRITELN('*************    END    *****************');
  WRITELN(FILER,'*************************************',
      '***************************************');
  CLOSE(FILER)
END. (** STOCHASTIC AVAILABILITY **)
```

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